6,000 Kva. Synchronous Phase Modifier and Exciter.

# CONSTANT-VOLTAGE TRANSMISSION

A Discussion of the Use of Synchronous Motors for Eliminating Variation in Voltage in Electric Power Systems

BY

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# PREFACE

THE advantages of constant-voltage transmission, in improved operation and lower cost, have been recently proved by actual examples, among them that of the longest transmission line yet built. The practical success of the method justifies the statement that the use of synchronous motors has been far too limited in the past.

It is the purpose of this book to urge that more synchronous motors be installed in alternating-current power systems, and that dependence be placed on them to secure the desirable results of controlling the voltage of lines at the opposite end to that of usual practice, and of more than doubling the power load of most lines. In this way, the installation of comparatively inexpensive machines can take the place of building entire duplicate transmission lines.

The decision regarding such important changes in design and operation, even when the examples described are kept in view, must be made according to thorough predeterminations of cost and operating characteristics. Working formulas, with examples, are given for these comparatively new calculations.

Although the writer is in favor of the increasing use of the principles of constant-voltage transmission, both in long-distance work and local distribution, he has tried to show impartially both sides of the case, and to outline the conditions where the new method is not applicable.

References and acknowledgments to other discussions of this subject will be found in Chapter VII.

H. B. DWIGHT.

HAMILTON, CANADA,
June, 1914.

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# CONSTANT-VOLTAGE TRANSMISSION

## CHAPTER I

### INTRODUCTION

THE new system of constant-voltage transmission has been successfully applied to both large and small power projects, and the results thereby obtained demonstrate convincingly that this method must be closely investigated by those responsible for the management or design of electric-power projects. The advantages of synchronous motors for raising the power-factor of the load of a steam-driven station have been thoroughly described. It is, however, not so well known that synchronous motors are of far greater value with long-distance transmission systems than with local power systems, so much so, in fact, that synchronous motors can often be installed to run idle, and yet save several times their cost.

The constant-voltage method depends on the use of synchronous motors, or phase modifiers, in large quantities in the receiving substations. The excitation of these machines is continually changed, either by hand control or by automatic regulators such as the Tirrill regulator, and by thus changing the power-factor, the voltage drop in the transmission line is held at a constant value. In this way voltage variation on the transmission line is eliminated, and this result is so valuable that the advantage of the synchronous machines for raising the load power-factor, while it exists, is by comparison, of secondary importance.

The valuable features of constant-voltage operation by means of power-factor adjustments consist in more than doubling the maximum power load of many existing lines, thus making fewer or cheaper transmission lines necessary for a given project. The voltage is controlled at the receiving end, which is a much more desirable place than at the supply end. The design of transmission lines may be affected, and heavier conductors, wider spacing, and higher reactance may be used than are generally found practicable. The economical distance of transmission is distinctly increased by the constant-voltage method. The adoption of this method of control for a transmission system will not lessen its reliability. An item of very practical importance is that the method can be gradually adopted without disturbing the normal operation.

A power line, as ordinarily operated, has at some part of its length a very noticeable variation in voltage from hour to hour. This is often the most troublesome feature encountered in delivering a large power load. Furthermore, since in most lines a steady voltage is desired at the receiver end while the adjustments in voltage must be made at the supply end where the receiver voltage cannot be observed, it is often extremely difficult to produce a satisfactorily steady voltage. With a constant-voltage line, on the other hand, there need be no voltage variation at either end, and it is far easier to obtain a steady voltage as shown by a voltmeter at any substation, because adjustments in voltage are always made within sight of the voltmeter in question. In other words, since steady voltage is required at the receiver end of a transmission line, that is the rational

place in which to make adjustments in voltage, either by hand or by an automatic regulator. This is done with the constant-voltage method, and, in addition, the same steady voltage is produced in the same way at all other parts of the line or system.

The usual limit to the amount of power which a given line can transmit is set by the voltage variation produced by a large power load. With the constant-voltage system, voltage variation is overcome, and the amount of power can be increased until some other limitation is encountered. This limitation will be imposed by the cost of power for extra line losses at the heavier load, or by the cost of the synchronous motors needed to maintain constant voltage. In either case, the power rating of a line will be found to be approximately twice as great as when synchronous motors are not used. If heavier conductors are used, which is often found to be economical with the constant-voltage system, the power rating of one line may be multiplied by three or more. With lines of moderate or great length the cost of the synchronous motors needed to double the rating of the line is a small fraction of the cost of an extra transmission line. Hence results the very great saving in money produced by the constant-voltage system, which is the strongest reason for the installation of sychronous motors in transmission systems.

A noticeable feature of modern practice is the increasing use of current-limiting reactances, in order to give greater reliability. Ordinarily, the chief obstacle to their use in transmission systems is the resulting extra voltage variation, but with the constant-voltage system this obstacle is overcome at small cost, as is shown by the

examples in later chapters. Although it is possible for sychrononous machines to drop out of step, this danger is much less with the latest types of machines, especially if they have no mechanical load, and very large proportions of synchronous machines are now successfully being used in the receiving stations of large transmission lines. Taking into account both of the above considerations, it may be stated that the constant-voltage method does not reduce the reliability of operation.

Perhaps the most important recent development in power transmission is the growth of extensive high tension net-works, which, having already stretched between cities nearly 800 miles apart, would seem to have no limit whatever to their growth. The economies and advantages of the constant-voltage method will assist the growth of such net-works.

Constant-voltage transmission is not merely a theory. It has already been utilized successfully in several power systems, both of large and small size. A brief outline of some of these, as taken from descriptions in the technical magazines, is given in Chapter VII.

It may be stated that without doubt not nearly enough synchronous machines are made use of at present for the best economy and the best operation. As there are attractive possibilities in constant-voltage transmission, both for long-distance projects and for city systems, and as there are also cases where the constant-voltage method is scarcely applicable, it is worth while obtaining a clear understanding of the principles of this method, since they differ quite widely, both in theory and practice, from the more usual methods.

# CHAPTER II

## ALTERNATING-CURRENT TRANSMISSION

ONE of the chief reasons why the electrical development of the past thirty years has been more rapid than has ever been the case with any other phase of engineering is that electricity offers the cheapest and most convenient means of supplying energy according to the various needs and occasions of the users. A consumer can withdraw from a common supply amounts of energy, either large or small, at any time and at almost any location, which he desires. In this quality of flexibility of application, electricity is superior to steam, gas, shafts, or ropes, for delivering energy.

The main difficulty connected with most methods of energy transmission is that the form in which the energy is most economically transmitted is quite different from the form in which it must be applied. This is true, also, of electric power, but, fortunately, the transformation from high voltage, which is most suitable for transmission, to low voltage for ordinary application, is very easy and inexpensive. An alternating-current transformer is one of the most efficient pieces of apparatus built, and one of the cheapest, considering the amount of power passing through it.

The design of electric circuits is largely determined by the distance between the point of supply and the point of application of energy. For short distances of only a few hundred feet, electric circuits for ordinary

applications usually have a voltage between conductors of 110 or 220 volts, since such a voltage is suitable for lamps, small motors, and many appliances. If the distance is increased, it is found that the voltage used up in driving the current through the long wires becomes a noticeable proportion of the working voltage, which is thereby subjected to troublesome variations, as described more fully in Chapter V. This trouble could be counteracted by using very large conductors, but it is cheaper to apply a higher voltage, such as 2,200 volts, to the circuit, and then transform to the low voltage where needed. With this arrangement, the voltage drop is a small percentage of the higher line voltage, and its actual value is reduced, since the current is less for a certain amount of power at the high voltage. It was early found in the development of electrical machinery that the use of high voltages was a very successful means of obtaining economical transmission of energy to long distances, and within a few years the voltages used had reached many thousand, and the distances traversed had extended to many miles.

It is obvious that for the longest distances, use must be made of the highest voltages in order to keep the current small and the voltage variation within reasonable limits. The voltage used for ordinary cases can be approximated by the rough rule of 1,000 volts per mile of distance of transmission. This represents the choice of a voltage giving a balance between the extra cost of heavy conductors when the voltage is low and the extra cost of transformers and circuit breakers, and of generally superior construction, when the voltage is high.

When the voltage is much over 110,000 volts, the cost

of the high-tension apparatus increases disproportionately fast, so that the voltage is seldom increased above this figure, even for very long lines. For such long lines, less than 1,000 volts per mile is usually used. In connection with this economical limit to the use of extremely high voltage, it may be noted that some large capacity, extra high voltage oil switches recently built occupied 1,200 cubic feet and weighed 14 tons each, and that the following comment on them was made in an editorial of *The Electrical World:* "The time seems almost to have arrived in high-tension work at which the generators and prime movers become comparatively insignificant." Another evidence of the extreme cost of very high voltage apparatus is given by the fact that some 5,800 Kva. 150,000 volt transformers could have been built, according to an article in *The Electric Journal* of August, 1913, for a rating of 30,000 Kva. at 22,000 volts, without increasing the size. Plainly, electrical engineers are able to utilize extremely high voltages, but the cost becomes excessive.

It has been shown that, as the distance of transmission is increased, it pays to increase the expenditure at the line terminals in order to decrease the size or number of the conductors. This is usually done by using higher voltage, and a limitation to this development is being felt in the rapid increase in cost of the terminal apparatus as the voltage is raised to an extreme amount. But this need not put the final limit on the distance of economical energy transmission, for the size of the conductors can be decreased, without raising the voltage, by installing synchronous phase modifiers and adopting the constant-voltage system of line control.

# CHAPTER III

## SYNCHRONOUS MOTORS

The constant-voltage system consists essentially in counteracting directly by means of synchronous motors the variation in voltage which is the most troublesome feature encountered in designing lines for the transmission of electric power. In order to show how the voltage variation may be overcome, a short description of the properties of a synchronous motor is necessary.

A synchronous motor is in itself practically a duplicate of an alternating-current generator. It, therefore, has definite poles and must rotate at the exact speed determined by the frequency of the power circuit to which it is connected. It also has its field excited by direct current, and the amount of the field current can be controlled by a field rheostat.

A reduction in the field current of a generator weakens the magnetism of the poles and lowers the voltage of the generator. However, weakening the field current of a synchronous motor cannot produce a proportionate decrease in voltage, because the motor is connected to an A. C. source of power of more or less constant voltage. It is found that when the field current of a synchronous motor is reduced, a lagging quadrature current flows from the A. C. line, which takes the place of the deficiency in field current in magnetizing the machine. This current is quite similar to the quadrature or reactive current drawn from the line to magnetize an induction

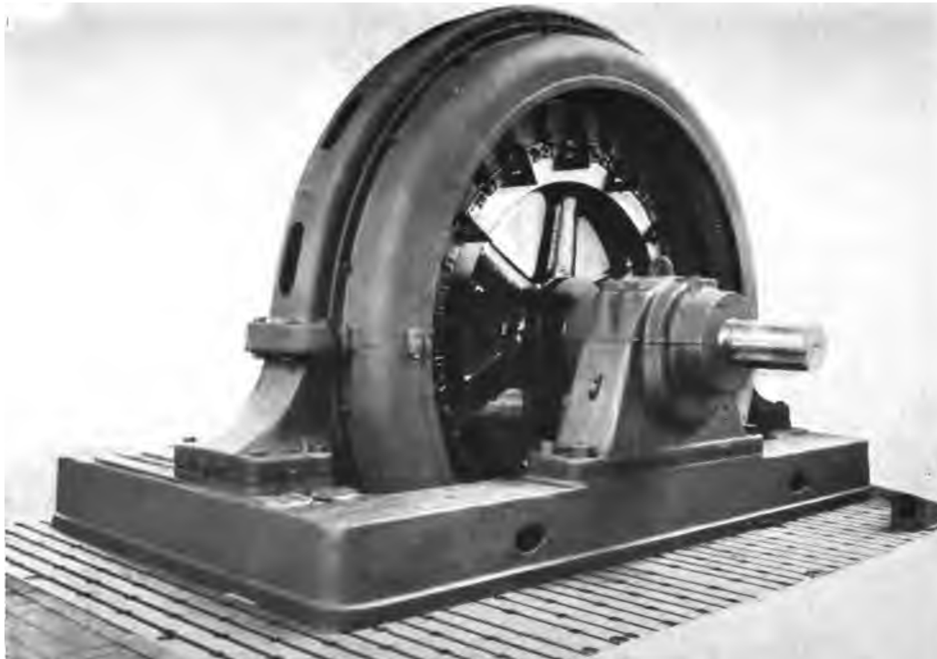

FIG. 1.—2,000 Kva. Synchronous Motor or Phase Modifier.

motor, and the nature of this current will be quite generally understood when it is stated that a synchronous motor with a weak field current has a similar effect on an A. C. line to the effect of a low power-factor induction motor. It may be mentioned that the lagging reactive current described in this paragraph has the usual effects noticed with low power-factor loads, of increased drop in the A. C. lines, increased current in the transformers and generators supplying the lines, and increased difficulty in holding up the voltage of the generators.

When a very large field current is used with a synchronous motor, a quadrature demagnetizing current flows from the line, so as to keep the magnetic field of the motor at the correct value for the line voltage. This current is, of course, directly opposite to the magnetizing current which flows when the motor is under-excited. It is, therefore, a leading current, and can counteract the bad effects of low power-factor induction motors connected to the system. It decreases the voltage drop in the line, and raises the power-factor at the generators.

When a reactance is connected across an A. C. line, the current which flows is called a lagging current, since it lags behind the line E. M. F. This current is the same as the current which flows when a synchronous motor with a weak field is connected across the line, So, also, when an ordinary condenser, built up of parallel plates, is connected across an A. C. line, the current leads the line E. M. F. and is called a leading current, and this is the same as the current which flows when a synchronous motor with a very strong field is connected across the line. To avoid confusion, in all cases in this book a synchronous motor will be spoken of as having a lagging

current when its field is weak, and a leading current when its field is strong. This should be clearly understood, because the notation used in this book is not used in all discussions of alternating currents, especially where more emphasis is placed on the action of currents and E. M. F.'s inside the machine than on what takes place in the A. C. line.

An explanation of the meaning of lagging and leading alternating currents, and of how the relation between voltages, currents, and power-factors may be expressed by vector diagrams when sine wave forms are assumed, will be found in text-books on alternating currents.

It is to be noted that a synchronous motor with a weak field current has a small pull-out torque and poor synchronizing power, and it will, therefore, more easily drop out of step at changes in its mechanical load or at disturbances in the line voltage than when it has a strong field current, when it is more stable and dependable

The changes in the armature current and the power-factor of a synchronous motor corresponding to changes in field current are shown by the well-known "V" curves, as in Fig. 2. The minimum value of armature current, between the lagging current and leading current branches of the curve, denotes that the motor is working at that particular stage at 100 per cent power-factor, that is, that the current supplied to the motor is in phase with the voltage and is all employed in doing useful work. It is evident from the "V" curve that a synchronous motor can deliver a certain amount of mechanical power with the least heating when it operates at 100 per cent power-factor; at either lagging or leading power-factor it must carry additional armature current, with correspondingly

greater heating. At leading power-factor, the heating for a certain current is greater than at lagging power-factor, due to the extra field current flowing in the motor.

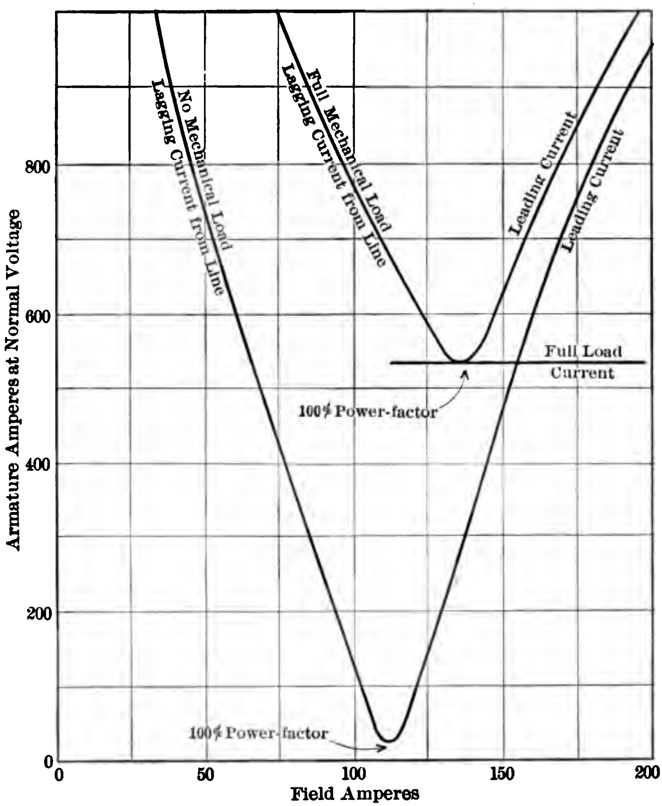


FIG. 2.—"V" Curves of a Synchronous Motor.

The property of synchronous motors of operating at leading power-factor with a strong field current is a valuable one and is very generally recognized. In taking advantage of it, the motor field current is generally left

at a fixed value. The other property, of giving easy adjustment to the power-factor, making it sometimes leading and sometimes lagging, is really distinct from the first property, and is of far greater value, as will be pointed out later. In order to make use of it, the extra trouble of making continual adjustment of the field current is necessary, but when the great advantages of continual adjustment of the power-factor are generally understood, the present common practice of leaving the synchronous motor field current at a fixed strength will be largely done away with.

An electric motor is a machine capable of absorbing electric energy and doing mechanical work, and synchronous motors are generally used for this purpose. They have also often been used in connection with steam-power plants, merely to be run idle with over-excited fields and to correct low power-factor of the load, thus reducing the current in the generators and feeders. When machines were designed for this purpose, economies were effected by using smaller bearings and omitting any mechanical connection such as pulley or coupling. Such special features of design and application warranted a distinctive name, and the name "synchronous condenser" was applied. This name is quite appropriate for the case just described, for with a strong field current, which is always necessary for raising the power-factor, the machine has the electrical characteristics of a condenser, since its armature current is leading.

The name "synchronous condenser," however, is not quite so appropriate when the machine is used with a constant-voltage transmission line, because for a large share of the time the current in the machine is not lead-

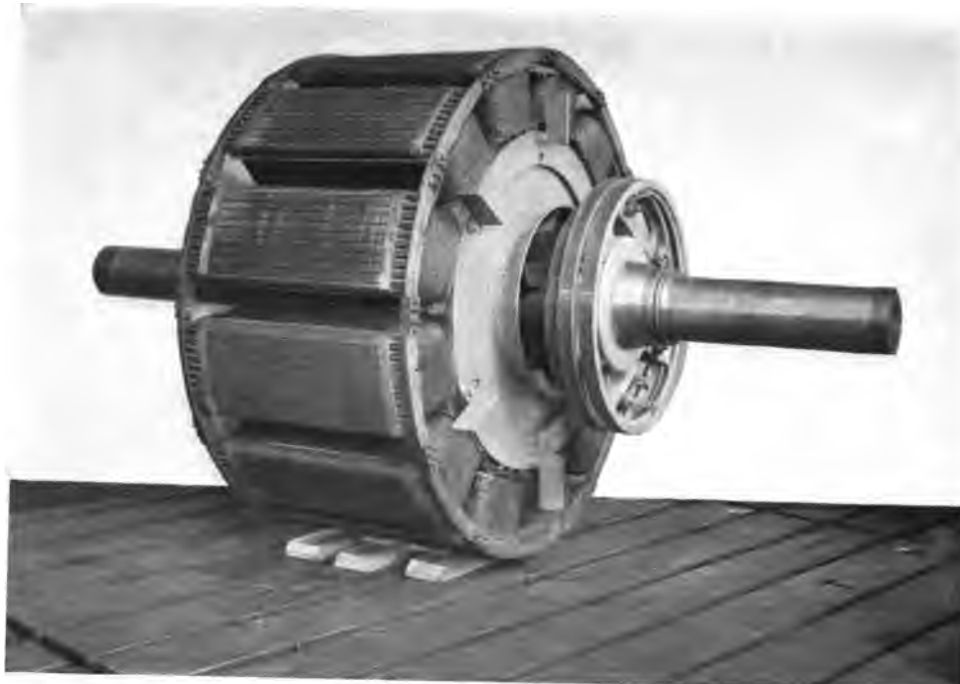

Fig. 3.—Rotor of Phase Modifier.

ing, but lagging, and the machine at that time does not behave as a condenser, but would more accurately be called a "synchronous reactor." Since the chief value of the machine in the constant-voltage system is not its ability to operate with leading current, but rather its ability to change from leading to lagging as desired, it would seem more appropriate to use the name "synchronous phase modifier," which is recommended in the "Standardization Rules of the American Institute of Electrical Engineers." In this book, all three names are used for a synchronous A. C. machine driven by alternating current, and no essential difference is intended when the different names are used.

One of the most severe handicaps in the commercial application of synchronous motors has always been the low starting effort developed by the machine, and the trouble in synchronizing it. The use of a cage winding on the field has greatly improved matters, so that now synchronous motors can develop about 30 per cent starting torque with one and one-half times full load current, and they synchronize themselves. The above starting torque is sufficient for a great many motor applications. In Fig. 3 is shown the rotor of the 6,000 Kva. phase modifier, illustrated in the frontispiece. The cage winding, consisting of copper bars in the pole face and heavy end rings, may be clearly seen.

## CHAPTER IV

### POWER-FACTOR CONTROL OF LINE VOLTAGE

THE most common means at present for making adjustments in line voltage so as to deliver steady voltage to customers consists in using a potential regulator. This consists generally of a low-voltage transformer in series with the line, so that it adds a certain amount of voltage directly to the line voltage. An electric relay operates a mechanical device which changes this increment of voltage in accordance with changes in the load and potential, in such a manner that a constant voltage is kept at the centre of load of the line. Potential regulators are exceedingly useful and reliable pieces of apparatus, and they are extensively used on the feeders of distribution systems. It is the purpose of this book, however, to discuss the fact that synchronous motors can also be used as potential regulators. They are not in very direct competition with the above-described transformer type of potential regulator, since the field of application of synchronous motors is with large blocks of power, and especially with transmission lines.

It is a very firmly entrenched idea, but correct only for certain applications, that synchronous motors should be used in alternating-current systems only in order to raise the power-factor of the load on the generators from a low to a high value, and that 100 per cent power-factor at all times is the ideal condition. Synchronous motors, when employed for this purpose, may be said to be used

as "power-factor correctors." This, however, is their most useful employment only in systems where the energy is all used very close to the generators. The early central station power systems were of this type. When their loads gradually changed from a high power-factor lighting load, to a low power-factor induction motor load, the troubles which they experienced from low power-factor, as described in Chapter IX, made a wide-spread impression, and determined to an excessive degree the method of looking at the application of synchronous motors.

In most systems, the transmission of the energy, even across a city, plays a very important part and involves a large share of the total expense. Synchronous motors should in such cases be used as "voltage regulators," since they can thus give commercial benefits, as described in Chapter VI, much in excess of the benefits from mere power-factor correction. The best part of the matter is that the benefits from power-factor correction are not sacrificed, but the synchronous motors fulfil both functions as "voltage regulators," and as "power-factor correctors" at times of heavy load when power-factor correction is most valuable.

The theory of how synchronous motors can be used as voltage regulators is easily understood by considering the vector diagram of an A. C. transmission or distribution line. See Fig. 4. This diagram is a very familiar one in A. C. work. Let $E_s$ be the voltage at the source of energy supply, and $E$ the voltage at the other end of the line where the energy is applied. Let the current $I$ be composed of two components, $P$ in phase with the voltage $E$, and $Q$ in quadrature with $E$ and lagging be-

hind it. Let $R$ be the resistance of one conductor, representing the loss of energy appearing as heat in the conductors, and let $X$ be the reactance of one conductor, representing a drop in voltage caused by the alternating magnetic field around the conductors. There will be a drop along the conductors proportional to the current and

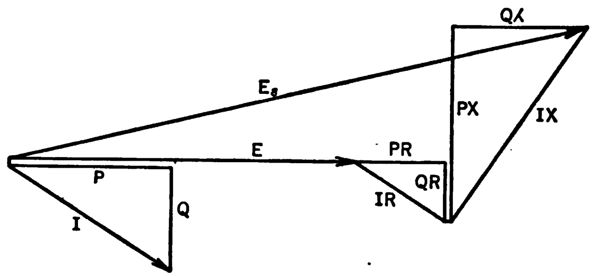


FIG. 4.—Vector Diagram of A. C. Line with Lagging Current, Conditions Given at Receiver End.

the resistance and reactance. The resistance drops, $P\,R$ and $Q\,R$ are in phase with their respective currents, and are drawn parallel to them. The reactance drops, $P\,X$ and $Q\,X$ are drawn at right angles to the corresponding currents. From the vector diagram, the supply voltage $E_s$ may be seen to be given by the equation

$$E_s^2 = (E + P\,R + Q\,X)^2 + (P\,X - Q\,R)^2$$

or approximately,

$$E_s = E + P\,R + Q\,X + \frac{(P\,X - Q\,R)^2}{2\,(E + P\,R + Q\,X)}$$

or, to a rough approximation,

$$E_s = E + P\,R + Q\,X$$

The voltage drop is thus seen to be

$$P\,R + Q\,X$$

and it is the increase and decrease of this quantity as the load changes which cause the troublesome variations in line voltage to which reference has previously been made.

It has been stated that by changing the excitation of a synchronous motor, a large lagging or leading quadrature current may be made to flow in the A. C. line which is supplying power to the synchronous motor. Thus, if the field current is weakened, $Q$, which is shown as a lagging current in the diagram, is increased. Similarly if the field current is strengthened, $Q$ is decreased, and may even be made negative, in which case it becomes a leading current. The voltage drop is now

$$PR - QX$$

that is, the leading current reduces the drop and so tends to increase the delivered voltage. Since reactance is always prominent in A. C. lines, it is plain that changes in $Q$ produced by altering the field current of a synchronous motor have a powerful effect in raising or lowering the line voltage at the synchronous motor. This may be tested by any one, by simply moving the field rheostat of a large synchronous motor. In many usual cases a variation of 10 per cent in the voltage at the motor may be produced in this way, since reactance in the transformers supplying the motor acts in the same way as reactance in an A. C. line. It is thus seen that by controlling the power-factor by means of synchronous motors, it is possible to obtain a very effective control of the voltage.

# CHAPTER V

## DISADVANTAGES OF VOLTAGE VARIATION

VOLTAGE variation has been stated in a previous paragraph to be one of the main features to be taken into account in designing an electric-power system. A short description will be given of some of the disadvantages of variation in the voltage.

Where electric power is being supplied to customers in a city, a large part of the power is used for lamps. These, perhaps more than any other type of commercial power-consuming apparatus, require a very constant voltage. If the voltage is lowered a small percentage below normal, the brightness of an incandescent lamp is decreased by a much greater percentage. If, on the other hand, poor line regulation causes a rise of voltage above normal late at night when the total load is low, the disadvantage is encountered that the lamp deteriorates at a rapid rate and its life is shortened. This is a matter of considerable commercial importance in connection with expensive modern metal-filament lamps. Mercury vapor lamps also require a voltage very close to that for which they were designed, for satisfactory operation. Even heating and cooking devices, such as are used on lighting circuits, operate best at their normal voltage, and separate types are manufactured for normal voltages of 105, 110, 115, etc. Since there will often be two or three volts drop in the local 110-volt circuit, it is evidently necessary to maintain practically constant

voltage in the 2,200-volt circuit which supplies a lighting load, and this is usually done by feeder potential regulators.

It is allowable to have somewhat larger variations in voltage on motor circuits than on lighting circuits. It is the usual practice to supply motor loads and lighting loads from separate local circuits, thus avoiding any noticeable change in brightness of the lamps when motors are started up, and allowing a smaller size of wire to be used for the motor circuits. But it is distinctly disadvantageous to have more than a small percentage of voltage variation even in a motor circuit. A motor operated at a voltage above normal has a saturated iron circuit, with consequently large magnetizing current and heating. If the voltage is lowered below normal, induction motors and synchronous motors are liable to stop or "pull out" under heavy loads. There is, also, often difficulty in starting them when the voltage is low, since both the starting torque and the pull-out torque are rapidly reduced with a decrease in voltage.

Let us now consider the disadvantages of voltage variation on transmission lines. Transmission lines generally carry power from a water-power plant to a city. As stated in the preceding paragraphs, the voltage of a city net-work must be maintained as close as possible to a constant value. But the voltage at the receiver end of a transmission line, that is, the end at the city, may sometimes be allowed to vary 5 or 10 per cent, if suitable feeder potential regulators are installed to correct for this variation. Feeder regulators can scarcely be expected to correct for more than the above amount of variation, since they must also compensate for the load drop

in the lines of the city net-work. Accordingly, if the transmission-line voltage at the city end has too great a variation for the capacity of the potential regulators, there will be variation in the voltage actually supplied to customers, and this will entail all the disadvantages in the use of electric power described above. Power companies depend, for increasing their power load, on keeping customers satisfied by giving them good service. Allowing noticeable variations in the voltage supply is one of the surest ways of making a power-user dissatisfied with the service of the supply company in particular, and with electric power in general. Moreover, as is often pointed out, allowing the voltage to fall by a certain percentage at peak load produces twice as great a percentage decrease in the power consumed by both lamps and motors. This is annoying to a customer, since he wishes full power and is willing to pay for it, but it is a direct money loss to the power company, whose income is based on the amount of energy registered by the meters.

In a great many power-transmission systems, energy must be sold for application close to the generating plant, and voltage variation at the end of the transmission line next the power plant has the same disadvantages as variation at the city end of the line. There are cases where there are no customers close to the generating station, and as far as the sale of power is concerned, the only place where steady voltage is desired is at the receiver end of the transmission line. The voltage of the generators may, therefore, be varied through a large range in order to compensate for drop in the line. What, then, is the limit to the voltage variation allowed at the generating end? It is found as a result of practi-

cal experience in increasing the load of transmission lines that good operation cannot be obtained when the variation in voltage from no load to full load is more than 20 or 25 per cent. A small percentage variation in the load, such as is liable to occur suddenly on any power system, should not cause a fluctuation in the delivered voltage sufficient to be a detriment to the service. It is found with a line whose total variation in voltage is as great as the percentages mentioned above, that ordinary changes in load produce noticeable changes in voltage at the load which are too rapid and too great to be satisfactorily corrected by either hand regulation or automatic regulators. In other words, the line has poor regulation.

In many systems, hand regulation of the generator voltage is used rather than automatic regulation. In such cases, practical difficulty is experienced in compensating for large gradual changes in load, where the line has a large percentage regulation. The operator in the generating station must estimate from the readings of his ammeters and power-factor meter what his voltmeter reading and his rheostat setting ought to be. The result of his efforts is frequently unsatisfactory to the station receiving power, and readjustments of the voltage can only be obtained, after considerable delay, by using the long-distance telephone.

The disadvantages and difficulties described above in operating a transmission line with a large voltage regulation between no load and full load have been apparently insurmountable in many cases. As a result, when the regulation, due to increases in load, has reached 20 or 25 per cent, or sometimes a smaller percentage, new du-

plicate transmission lines have generally been built, thus adding greatly to the capital investment for the power system. The possibility of avoiding this expense by means of an automatic regulator in the generating station which would be capable of compensating for an extremely large line drop is discussed in Chapter XI, where it is shown that, even if such a regulator were produced, the resulting low line efficiency and low generator power-factor would make it inadvisable. Practical proof, however, has been given that it is commercially feasible to increase by a large proportion the power capacity of transmission lines by overcoming voltage variation by means of synchronous motors.

# CHAPTER VI

## ADVANTAGES AND DISADVANTAGES OF THE CONSTANT-VOLTAGE SYSTEM

Since voltage variation imposes the limit of the load of a transmission line as ordinarily operated according to the varying-voltage method of control, lines are generally designed, for the sake of economy, to have as great a voltage variation as is allowable. Therefore, when the load is as large as was planned, the voltage variation is often found to be the most troublesome operating feature. Other troubles, as from lightning, may be more serious, but they are occasional; voltage variation is always present. Thus it is obvious that by installing synchronous motors, or phase modifiers, and so practically doing away with voltage variation, the operation of the power system is made more easy and satisfactory. Moreover, it is possible to furnish a steadier voltage to all the customers than is often obtainable where the voltage is adjusted only at the generators. This results in greater economy of operation and less damage from excessive voltage to all kinds of electrical apparatus. There is, consequently, greater satisfaction with the power service, and power is easier to sell. Also, since the voltage does not drop on the occurrence of peaks in the load, more energy is registered by the meters and the income is increased. The operating advantages of constant voltage are, therefore, not without a definite cash value.

In water-power plants of moderate size, sudden changes of load not only produce the variations in voltage which have been described, but they alter the speed of the water-wheels during the intervals of time before the water-wheel governors can act. These changes in frequency aggravate the voltage variation, and are detrimental to the satisfactory operation of motors. A large flywheel effect of the system tends to minimize the variations in frequency. Synchronous motors add their fly-wheel effect directly to that of the system, and in this way they improve the regulation of both frequency and voltage of medium-sized power systems.

The greatest advantage of the constant-voltage system is, as has been mentioned, that it allows the load of a transmission line to be made two or more times as great as could ordinarily be carried, thus greatly reducing line costs. Where the limit of the load is set by the voltage variation, the efficiency is usually higher than would be needed for economic reasons. When voltage variation is eliminated by means of voltage control by synchronous motors, the load can be increased, and the limit that will be met in most lines will be the cost of power for extra line losses, since the efficiency of a transmission line is always reduced as the load is increased. If we assume that the load of a varying-voltage line is limited by a regulation of 15 per cent due to line impedance only, and that the load of a constant-voltage line is limited by an efficiency of 85 per cent, then the relative power capacity of lines controlled according to the two methods can be compared, as in Figs. 5 and 6.*

* See "Constant Voltage Transmission," Proc. A. I. E. E., June, 1913, p. 1362.

I. Power rating of constant-voltage transmission lines.

II. Synchronous phase modifiers required for the above constant-voltage lines.

III. Power rating of varying-voltage transmission lines.

The data for Figs. 5 and 6 are: Length of lines, 100

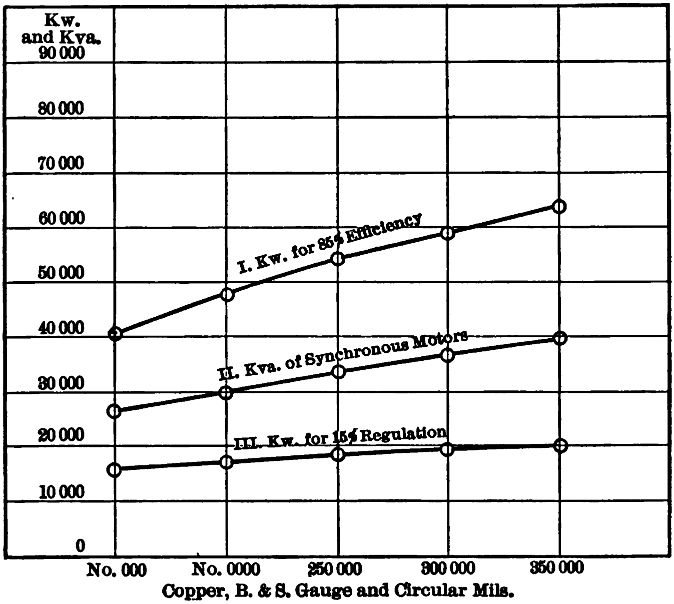


FIG. 5.—60 Cycle Lines.

miles; highest working voltage on lines, 115,000; power-factor of loads, 80 per cent.

The statement has been made several times in the preceding chapters that adjustable power-factor is much more valuable in saving line costs than mere high power-factor. In other words, the power rating of a line

is increased much more when a synchronous motor is used with continual adjustments in the excitation from leading at heavy load to lagging at light load, than when the synchronous motor is used with fixed rheostat position. Using the same limits for the power capacity of a line as were used for Figs. 5 and 6, a little calculation will

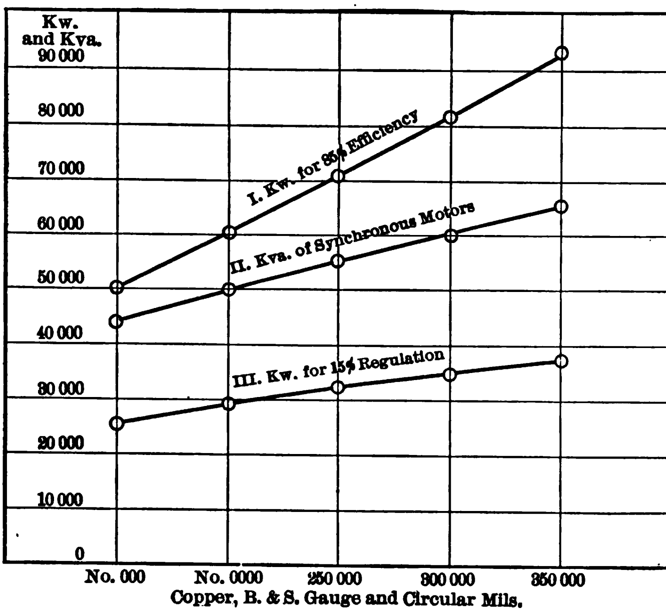


FIG. 6.—25 Cycle Lines.

show that by installing synchronous motors, with fixed rheostat position, of a rating equal to 42 per cent of the load kilowatts, the power-factor can be raised from 80 up to 95 per cent, and the kilowatt capacity of the line will be increased 54 per cent. Now if the motors are used for adjustable power-factor, and a somewhat larger

proportion is installed equal to 63 per cent of the load kilowatts, not only is constant voltage maintained, but the kilowatt capacity of the line is multiplied by 2¾, the efficiency of the line being lowered to 85 per cent.

This is summarized in the following table. Transmission line data: Length, 100 miles; No. 0000 copper cables; 10-ft. spacing, 100,000 volts, 60 cycles.

| Corrected P. F. of Load for Varying-voltage Line | Kw. of Load | % Kw. | % Regn. | Syn. Motors, Kva. | Syn. Motors, % of Kw. Load |
|---|---|---|---|---|---|
| 80 | 17,000 | 100 | 15 | None | 0 |
| 85 | 20,000 | 113 | 15 | 2,600 | 13 |
| 90 | 23,000 | 130 | 15 | 6,000 | 27 |
| 95 | 27,000 | 154 | 15 | 11,200 | 42 |
| Constant-voltage line at 85% efficiency...... | 48,000 | 277 | None | 30,000 | 63 |

Of the investment in power transmission projects, line costs form an important part, which increases as the distance of transmission becomes greater. Accordingly, it is found necessary with long distances to reduce the line costs by using higher voltages, although this increases the cost of transformers, circuit breakers, and substations. An economic balance is found in practice by adopting a voltage of approximately 1,000 volts per mile. A similar reduction in line costs may be made by installing synchronous phase modifiers and adopting the constant-voltage method of control. The adoption of this method may be equivalent, in reducing the cost of the transmission lines for a given amount of power, to raising the voltage approximately 50 per cent.

The phase modifiers will be most applicable where the voltage cannot be increased. The most common

case is where the expense of rebuilding existing parts of the line prohibits the use of as high a voltage as would be used for a new project. If, for example, it is desired to extend a 60,000-volt, 60-mile line 20 miles farther, it can be done more cheaply and quickly by installing phase modifiers than by building a higher voltage line the entire distance. This is also true at much lower voltages and shorter distances. In the same way, the use of phase modifiers enables overloads to be carried by transmission lines without impairing the quality of service, though the efficiency is reduced. On the other hand, if phase modifiers are not used, overloads mean either that the voltage variation is increased, thus giving poorer service, or else duplicate lines must be installed.

In city work, the voltage is limited, and the cost of lines is sometimes excessive, due to voltage variation, though heating of the conductors is often the factor to be reckoned with.

In long-distance transmission, the power loss at very high voltage due to corona, that is, electric discharge through the air between the conductors, puts a limit on the voltage. (See Table V.) An extreme case of this has been described by L. P. Jorgensen,* and it forms an interesting example of how the difficulty of a necessarily low transmission voltage was overcome at small expense by the constant-voltage method. Quoting from his description, "The lowest point on the line is the power-house, located at 12,200 feet above sea level, and the highest substation is located at nearly 16,000 feet elevation. The line runs for the greater part of

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* Proc. A. I. E. E., p. 510, March, 1914.

the distance of 120 miles over a comparatively flat plateau of 14,000 feet elevation. This place is called the roof of the world, and is up in the Andes Mountains, in Peru. The load on this system is mostly motors driving copper-mining machinery to the extent of about 10,000 horse-power, requiring a conductor of No. 1 size cable. The motors are divided up between synchronous and induction in such a way that the power-factor will be under complete control by means of automatic voltage regulators adjusting the excitation of the synchronous motors." The description further states that owing to the small size of conductor and the great altitude, the phenomenon of corona loss prohibited the use of a higher voltage than 50,000 volts, which is seen to be in agreement with Table V. As 50,000 volts is a very low voltage for a 120-mile transmission line, the above case is an unusually favorable one for the constant-voltage method.

Wherever there is a large number of scattered customers, each requiring a substation on the transmission line or net-work, the reluctance to using very high voltage will also be apparent. A 60,000-volt substation costs very much less than one designed for 100,000 or 150,000 volts.

For detailed examples showing the savings in cost, at a certain voltage, made by installing synchronous phase modifiers, see Chapter X.

The increase in carrying capacity of a transmission line due to the use of phase modifiers is greatest for large sizes of conductors, as is shown by an inspection of Figs. 5 and 6. With varying voltage, it is not economical to use a larger size of conductor than about No. 0000 cop-

per, since doing so would not make more than a small improvement in the regulation. The reason for this is that the regulation, $P R + Q X$, depends mainly on the reactance $X$, which is several times larger than the resistance $R$, for the larger transmission lines, and the reactance is practically the same for different sizes of conductors. On the other hand, with constant-voltage operation the maximum load is determined generally by the efficiency, and so it depends more on the resistance $R$. Greater loads can, therefore, be carried by constant-voltage lines made up of larger conductors. It will be economical where large blocks of power are to be transmitted, to make use of conductors as large as 350,000 or 400,000 circular mils, copper, instead of building several separate tower lines with small conductors, as is necessary with the usual varying-voltage method of control. It may be noted that still larger sizes of conductors than those mentioned would not generally be economical, owing to the theoretical limit of the load of a line, which is only encountered with extremely large conductors, and which is described in Chapter XII. When this limit is approached, the number of phase modifiers required becomes excessively great. It may thus be stated that one advantage of the constant-voltage system is that it allows the use of large conductors, which increases considerably the power capacity of a single line, and so produces additional savings in the cost of towers and land.

Since high reactance is not a very great disadvantage in designing a constant-voltage line, and is in some ways an advantage, since it reduces short-circuit currents, a very wide spacing between conductors may be used.

This, together with a large diameter of conductors, very considerably reduces the possibility of corona loss. Thus a line designed to carry as much power as possible, according to the constant-voltage system, may be operated at a considerably higher voltage than one of more usual construction, owing to the limitation of voltage imposed on the latter line by the phenomenon of corona. For example, No. 0000 cable at 12 feet spacing has a corona limit of voltage of 135,000 volts, while 300,000 circular mil cable at 20 feet spacing has a limit of 171,000 volts. See Table V.

Thus the constant-voltage system, by increasing the power rating of a given line, and by allowing the use of larger conductors and of higher voltages, extends to a marked degree the distance to which electric power may economically be transmitted, since this depends largely on the line cost for a certain amount of power.

Another advantage of the constant-voltage method of control is that it encourages the use of high reactance in all the different kinds of apparatus making up the transmission system. Without power-factor control, a line has its power-rating directly reduced when the reactance of any part is increased, since the voltage variation, which determines the power-rating of the line, is thereby increased. But the addition of reactance has a less effect on the power-rating of a constant-voltage line. This has been already mentioned in connection with the high reactance resulting from wide spacing of the conductors. The same fact is true as regards increasing the reactance of the transformers and generators, and especially as regards using protective coils of very high reactance. The value of high reactance in all parts of a

power system is well known, and consists in the reduction of short-circuit currents. Where a large number of generators are connected in parallel, the current flowing when a short circuit occurs is enormous, and has great destructive effects in distorting the ends of generator coils, in wrecking transformers, and in blowing up circuit breakers. The value of this current may be calculated,* and it depends on the amount of reactance in its path. It has been shown by oscilligraph records, and proved by the resulting immunity from accidents, that high reactances give valuable protection when short circuits occur.

One of the main reasons for the adoption of the frequency of 25 cycles to any great extent in this country was because the low reactance pertaining to it allowed larger loads to be transmitted than could be done with the same voltage variation at the higher frequency of 60 cycles. In other words, line costs were lower at 25 cycles. Other reasons for the lower frequency were sometimes the inherently slow speed of low head water wheels, which made the 25-cycle generators more economical, and often the desirability of using 25-cycle current for synchronous converters. The last reason is disappearing of late years, since 60-cycle synchronous converters are now manufactured which give practically as reliable and satisfactory operation as 25-cycle converters. With the constant-voltage system of transmission, the first reason for low frequency also largely disappears, because low reactance, as stated above, is scarcely an advantage with this new method of transmission. It

* See "Current Limiting Reactances," Proc. A. I. E. E., February, 1914.

would be very desirable to standardize on one frequency for electric-power apparatus, and if this were done in America, there is no doubt that the frequency would be 60 cycles. This frequency is by far the most popular for the supply of power to towns and cities. Generators, transformers, and motors generally cost less when built for the higher frequency, and most lighting devices, especially tungsten lamps and arc lamps, give much more satisfactory results at 60 cycles than at 25 cycles. Perhaps the most important class of load which must inherently use low-frequency current is that of alternating-current railways. But the fact that the load is often single-phase, and that the momentary peaks of load are very great, being sometimes of the order of 5,000 kilowatts or more, would render advisable the use of A. C. motor-generator sets, even when the 25-cycle load is to be supplied from a 25-cycle high-tension power net-work serving all classes of loads in the vicinity. If A. C. motor-generator sets must be used, it is obvious that a 25-cycle high-tension net-work would have no advantage over a 60-cycle one in supplying a 25-cycle railway. Thus it is seen that the standardization of the frequency of 60 cycles for transmission-line systems is desirable and possible, and it would be greatly assisted by the adoption of the principles of constant-voltage transmission.

A very attractive and practical feature of the constant-voltage system is that any existing transmission line or net-work can be gradually and easily changed over so as to operate, completely or in part, according to the new method. For example, consider a varying-voltage line which has 20 per cent regulation at the

generator end and steady voltage at the receiver end. By installing a small number of synchronous motors at the receiver, and continually adjusting them to give as constant voltage as possible, the generators may be operated with only 10 per cent voltage variation. The line is thus midway between a varying-voltage line and a constant-voltage line, since twice the number of synchronous motors would hold the voltage steady at both ends of the line. Thus, without making any changes in the construction of the line itself, it is possible to install a single small synchronous phase modifier and obtain a proportionate improvement in closer regulation or in increased carrying capacity. The operation of this first unit can be observed and its value and reliability thoroughly tested before investing the capital needed to change over completely to constant-voltage operation. On this account, then, the constant-voltage method should commend itself to any power system which is already built and whose load is large or is growing, since the adoption of the new method does not necessitate any interruption to service, nor any considerable risk as to investment of capital or reliability of the service.

In order to make the description of the constant-voltage system complete, its disadvantages should be described. First, as to reliability of operation, it is well known that large synchronous motors are not always considered to be reliable in connection with transmission lines, owing to the danger of their dropping out of step. This might produce large accidental variations in voltage, since the voltage of the transmission line is dependent on the synchronous machines. However, modern

synchronous motors show small tendency to be unstable, and the phase modifiers, not being connected to any mechanical load, would probably be especially safe, and would remain in step at times of line disturbances as tenaciously as a fully loaded induction motor.

Another disadvantage of synchronous machines is that they tend to increase the current flowing into a short circuit. In connection with this, it must be remembered that the synchronous phase modifiers are scattered, and are a long distance from the generators. The most dangerous short circuit is one close to the largest station, and synchronous machines at the other end of a transmission line cannot increase the short-circuit current appreciably, owing to the large intervening line reactance. Moreover, the increase in reliability due to the greater reactance in all parts of the system, made economical by the constant-voltage method, is probably of more effect than the above-mentioned tendencies.

A large number of lines is always useful, since it insures a good reserve in case of line trouble. The number of lines for any project must be reduced if the economies of the constant-voltage system are to be fully utilized. On the other hand, reserve is becoming of somewhat less importance because of the tendency to interconnect power systems as much as possible, and even to exchange power between independent companies. Reserve is also provided and extra line construction saved, by steam turbo-generators used as synchronous motors maintaining constant line voltage, and also used as reserve generators.

In view of the advantages described in this chapter of a system which is comparatively new as far as regards commercial applications, it may be stated as a general conclusion that not enough synchronous motors are in use and that their application could well be increased to a great extent. The right place to control the voltage of a transmission line or net-work is at the receiving stations. This must be done by synchronous machines, and the help of customers may be enlisted for attaining this result. The tendency of transmission-line design should, therefore, be to build lines of heavier conductors, at lower voltages, or for longer distances, and at the frequency of 60 cycles, and to hold the voltage constant at all points by means of synchronous phase modifiers.

## SUMMARY OF CHAPTER VI

Advantages of constant-voltage transmission:

1. Steady voltage.
2. Steadier frequency in small water-power plants.
3. Lower total cost where voltage is less than about 1,000 volts per mile. The low voltage per mile may be caused by extensions, city conditions, corona, or a large number of substations.
4. Overloads are better handled.
5. Large conductors reduce cost of towers and land.
6. Wide spacings and large conductors extend corona limit.
7. Increase in economical distance of transmission.
8. Better protection, due to high reactance.
9. Tendency to use 60 cycles.
10. The method can be gradually adopted.

Disadvantages:

1. Tendency of synchronous machines to drop out of step.

2. Possible large accidental variations in voltage.

3. Increase in short-circuit current, unless reactance is increased.

4. Decrease in reserve, due to fewer lines.

5. Higher total cost for short, high-voltage lines.

6. Lower efficiency.

# CHAPTER VII

## HISTORY OF THE CONSTANT-VOLTAGE SYSTEM

THE information perhaps most desired by any one contemplating the adoption of the constant-voltage method of control is a description of power systems which have actually used synchronous motors for the sake of the large economies described and have obtained satisfactory and reliable operation. Power systems must be excluded from this list of examples if they merely employ large numbers of synchronous motors with fixed rheostat setting, but do not continually adjust the excitation so as to overcome directly, by these adjustments, the variations in line voltage from hour to hour. While the list of veritable examples of power systems using the principles of constant-voltage transmission is not long, it is sufficiently large to show that the method is not merely theoretical, but has been proved in actual practice to be commercially economical and advisable under correct conditions.

The information in this chapter regarding actual instances of the use of the constant-voltage method, and references to the method by technical writers, is not complete, and is probably not in correct chronological order, since it is based merely on various items appearing in the electrical literature of the last few years. It will be observed that at different times in the past have the possibility and the profit of constant-voltage operation of transmission lines been suggested, though prob-

ably without stating quantitatively the large proportionate economy. So also, synchronous motors have been used in the past for overcoming line regulation by their frequent adjustment, and probably the automatic voltage regulator has contributed considerably to their success in this application. The writer believes, however, that only within the last year or two have the principles of constant-voltage transmission been urged as being immediately commercially profitable on a large scale, and only within the same recent period have transmission lines been constructed which not only would not carry their rated load without synchronous phase modifiers, but which have such heavy conductors and high reactance that they would be utterly uneconomical if operated according to any other method than the constant-voltage method.

One of the first enunciations of the constant-voltage method was very clear and definite. In a paper by B. G. Lamme entitled "Synchronous Motors for Regulation of Power-Factor and Line Pressure," Transactions A. I. E. E., 1904, page 481, synchronous motors were advocated, not only for changing the power-factor of a system, but for regulating the voltage and increasing the output of a transmission line.

In the discussion of the above paper, F. O. Blackwell referred to a 6,000-horse-power plant in India which transmitted power ninety miles, and where, by installing a 1,000-Kw. rotary condenser, 50 per cent more power was transmitted over the existing line, and the building of an extra transmission line, which had been contemplated, was postponed.

In the same discussion, W. L. Waters referred to the

value of a synchronous motor as a pressure regulator both with and without hand adjustment. F. A. C. Perrine pointed out that Tirrill regulators might be used, and further stated: "When we get away from the difficulty and the inaccuracy of hand regulation, we have overcome the most serious objection that has been raised in any of the discussions on the employment of the synchronous condenser."

About the same time, Dr. Steinmetz stated, in his book on electrical engineering, the possibility of holding the voltage of a transmission line constant by means of synchronous motors, and he gave a formula for the theoretical maximum power load of such a line, neglecting electrostatic capacity.

In February, 1911, R. A. Philip presented a paper before the A. I. E. E., and called attention to the theoretical possibilities of synchronous motors in aiding the growth of net-works. He also described the circle diagram of a constant-voltage line when the voltages at both ends were equal, and when electrostatic capacity was neglected.

In the Proceedings of the A. I. E. E., October, 1912, a description was given by P. M. Downing of the large 60-cycle net-work of the Pacific Gas and Electric Company, in the district surrounding San Francisco. This net-work consisted of 1,500 miles of 60,000- and 100,000-volt lines. It was stated that in some of the cities of this net-work, synchronous condensers, controlled automatically, were installed solely for voltage-regulating purposes. Steam turbo-generators, installed as reserves, were also used, running idle, for this purpose. As a result, the voltage regulation was greatly improved.

In the Proceedings of the A. I. E. E. for February, 1914, a report on transmission lines by P. M. Downing stated that synchronous condensers, operated as above, were giving complete satisfaction.

In December, 1912, an article by Lee Hagood in *The General Electric Review*, on "The Operation of Synchronous Machines in Parallel," stated very clearly the operating advantages to be obtained from using synchronous motors with automatic voltage regulators, and gave remarkably complete and convincing data from the Utica Gas and Electric Company, of New York State. The characteristics of the transmission system were: Length, 40 miles, 22,000 volts, 60 cycles, 13,000 Kw. of generators, and 3,200 Kva. of synchronous phase modifiers. The cost of the lines of this small system was not discussed, but many graphic voltmeter records were given to show that the improvement in operating conditions justified the expense of the synchronous machines.

In "Transmission Line Formulas," published by the author in February, 1913, a solution was given on page 40 for a constant-voltage line, allowance being made for electrostatic capacity.

In the Proceedings of the A. I. E. E. for March, 1913, a paper by L. B. Andrus described the benefits obtained from synchronous motors with automatic voltage regulators in the system of the Indiana and Michigan Electric Company. The voltage of this system is 25,000, and the frequency, 60 cycles. Synchronous phase modifiers, controlled by automatic voltage regulators, are used to hold the voltage at the receiver stations constant. The distance of transmission is about fifty miles. Savings in

line costs were not mentioned in the paper, but a clear case was made for the improvement in operation.

In an article in June, 1913, in *The General Electric Review*, on "The Operation of High-Voltage Power Systems," by H. H. Dewey, a description was given of the system of the Utah Power and Light Company, then under construction. The length is 140 miles; total power capacity, 42,000 Kw.; voltage, 130,000; and frequency 60 cycles. Two 7,500 Kva. synchronous phase modifiers were to be installed at the receiver station and to be controlled by automatic voltage regulators. They are of sufficient size to hold the voltage constant for loads from 5,000 Kw. to 38,000 Kw., but for larger or smaller loads than these limits the voltage must be varied. This line, like the Big Creek line, is an example of a case where absolute dependence is placed on the synchronous phase modifiers in order to deliver, for commercial use, the load planned for the line. In other words, the principles of constant-voltage transmission saved, in this case, the building of a duplicate 140-mile transmission line. It is typical of the way in which the principles of this new method of constant-voltage operation are misunderstood that the above article describing the Utah Power and Light Company stated, "The use of this synchronous condenser does not allow any saving in copper in such a line," while, as a matter of fact, the synchronous condensers had saved an amount of copper equal to the entire weight of copper used in the high-tension system.

In November, 1913, the longest straightaway transmission line in the world was put in operation, its length being 241 miles, from Big Creek to Los Angeles. In

January, 1914, the exact details of the line construction were published in *The Electrical World*. The data regarding the construction details of the transmission line of Example 1, Chapter X, are the same as for the Big Creek line. The data of that example regarding voltages and loading, etc., are merely hypothetical. It may be noted that the voltage is 150,000 and the power capacity of one circuit has been announced to be 60,000 Kva. It was at once obvious from the large size of conductor and the wide spacing, which gave a reactance more than four times the resistance, that dependence was to be placed on power-factor control in order to transmit enough power to utilize properly such large conductors. Synchronous phase modifiers of a total rating of 30,000 Kva., and controlled by Tirrill regulators, were installed from the first in the Los Angeles receiving station, and were sufficient to double the power capacity of the two 3-phase circuits. It was not altogether a surprise, therefore, when in *The Electrical World* of February 28, 1914, the engineering corporation responsible for the design and construction published the statement that "the condensers are used because without them the cost of the line would have been greatly increased to give the degree of regulation absolutely necessary for proper commercial service, . . . and the condensers are included as an integral part of the design, because economy has been obtained in the design as a whole through their use."

The Big Creek project may be considered to be the most striking and convincing example to which reference can be made, of the economy and advisability of constant-voltage transmission.

In March, 1914, the city of Winnipeg, Canada, decided to install two 6,000-Kva. synchronous phase modifiers, to be controlled by automatic voltage regulators, in order that they could increase the power capacity of their lines to take care of a rapidly growing load, without going to the much greater expense of building duplicate transmission lines. The distance of transmission is 77 miles, the voltage 66,000, and the frequency 60 cycles.

It is desired to refer, also, to the following papers by the author, which have presented many of the facts described in the present work:

"Constant Voltage Transmission," Proc. A. I. E. E., p. 1359, June, 1913.

"Maximum Loads of Transmission Lines," *The Electric Journal*, p. 838, September, 1913.

"The Use of Synchronous Condensers with Transmission Lines," Trans. Canadian Society of Civil Engineers, November, 1913.

"A New Principle in Transmission," *The Electrical News*, February 15, 1914.

"The Economies of Power-Factor Adjustment," *The Electrical World*, March 28, 1914.

# CHAPTER VIII

## HIGH-TENSION NET-WORKS

ONE of the most important tendencies of electric-power transmission at the present time is to connect numbers of transmission lines into large high-tension net-works. The Southern Power Company, of North and South Carolina, and neighboring companies with which it is interconnected, making over 1,000 miles of 100,000-volt lines connected together, and reaching across nearly 800 miles of country, and the Pacific Gas and Electric Company, of California, with a net-work of over 1,500 miles of 60,000- and 100,000-volt lines, are conspicuous examples. The tendency is exhibited in all parts of the country by the steady growth of net-works, and by the unification of different transmission lines and net-works into large power systems. A short discussion will be given of the causes for the growth of power net-works, and the relation thereto of the principles of the constant-voltage method of control.

The unification of the power systems of a section of country offers large financial rewards. Such a combination has probably more attractive possibilities of gain than are contained in the formation of the usual large industrial combinations, as of transportation or manufacturing companies, because in addition to the usual commercial advantages, the union of power systems has large engineering advantages.

Any large combination of industrial concerns is able

to carry on its business better than its smaller rivals in the following ways: It can bring the power of a large amount of capital to bear when it has any important undertaking to accomplish, such as making an extension; it can command better prices from other companies, since it handles a large volume of business; and it can afford to employ the most skilful and experienced managers, and all departments can be made to operate with the highest efficiency, not only when considered separately, but in their relation to the entire concern.

In the case of a large power net-work, there are, in addition to the above advantages, operating economies and electrical advantages of large importance. A net-work of long transmission lines generally receives the bulk of its energy from water-power plants. Now the ratio of the amount of energy actually paid for during a year, to the amount which a water-power plant could furnish if all parts of the system were operating at maximum load all the time, is usually exceedingly small. The cause for this is that there are great irregularities in the conditions of the application of electric energy and also of its generation by water-power. First, the demands for energy by the customers are very small at certain times, and again they are very large at certain seasons and certain times of day. Of course, if the energy-supply company gives good service, it must have a power supply and an entire electrical equipment large enough to take care of the maximum peak of the load. Secondly, the supply of water is nearly always irregular throughout the year. The amount of connected load which a water-power can supply is limited by the minimum rate of flow during the low-water season, and for a

large part of the year there is more water-power available than can be used.

The above conditions necessitate a large capital investment in order to supply energy to a certain number of customers, and such conditions are the cause of the unprofitableness of many water-power plants. They can be overcome to a marked degree by allowing the power plants to supply energy into a huge net-work by which electric energy can travel here or there over many miles of country, as the load demands, instead of each plant supplying its own small group of customers. The peak loads of different customers and different towns occur at different times of day. The load curves of a residence city and a factory town are quite unlike, and large differences also occur where two cities use standard time differing by one hour. The net-work averages up these irregularities in the demands for power, thus making a smaller equipment necessary for a certain number of horse-power of connected load, and, therefore, for a certain income from the sale of energy. This is generally expressed by saying that the load factor of a large net-work is higher than the load factor of a small system, the load factor being the ratio of the average load to the peak load. In the same way, the net-work averages up the irregularities in the water supply of different plants, and minimizes the necessity for water storage and for reserve equipment of all kinds in case of breakdown. Thus each plant may be designed to operate almost steadily at its normal load, since the net-work will take energy as the plant finds it convenient to supply it, nor is the net-work inconvenienced if a single plant shuts down. This results also in less power being wasted at times of

high water. That there is not the slightest doubt of great economies resulting from the formation of large net-works is proved by the fact that in many places competing power companies have made contracts with each other for the interconnection of their net-works and the exchange of power.

A constant-voltage straight transmission line will require synchronous motors of a total rating equal to about 60 per cent of the total power capacity of the line, as indicated by the examples in Chapter X. A constant-voltage net-work, however, will not require such a large proportion of synchronous motors, unless the generating stations are spaced very far apart. In a net-work, the amount of real power furnished by each generator is not controlled by electrical adjustments, but is determined almost entirely by the setting of the governor of each prime mover. The governor will be set so that there is a certain tendency to increase the frequency of the entire system, which means that the generator is delivering real power to the system. A change in the generator rheostat changes the power-factor at the generator, the amount of reactive power which it delivers to the system, and indirectly the voltage. Thus a small generator in a net-work provides exactly the same means of maintaining constant voltage where it is located as a synchronous motor in a substation, and so if there are a number of scattered generators in a net-work, the proportion of synchronous motors will be expected to be less than 60 per cent, even when the lines are loaded to a maximum degree.

A small generator connected to a net-work can probably furnish the required leading and lagging reactive

Kva. more economically than a synchronous phase modifier, because it can at the same time furnish real power, and the real and reactive power will not be added directly to make up the load of the generator, but only added in quadrature. Thus, 70 per cent of reactive power and 70 per cent of real power make up only 100 per cent load on the generator. There will be small liability of trouble at times of light load when weak field current is used, because at that time the governor setting will be changed, or power will be completely cut off from the prime mover. For this purpose, an operator called a "load despatcher" is found necessary. A local operator can tell from his voltage how much reactive power his generators ought to furnish to the system and what his rheostat settings should be—he has merely to adjust for constant voltage,—but he has only slight information regarding how much real power he should furnish, and he must be informed as to this by a central operator who watches the load at all parts of the net-work, and who gives orders regarding the setting of governors, the starting and stopping of prime movers, and the connecting of transmission lines. The load despatcher has great opportunities of utilizing the water storage and other characteristics of different plants to the best advantage, and of avoiding the operation of machines at low load, and therefore low efficiency. He can also prevent the overloading of any connecting link in the network.

A synchronous motor can carry real and reactive loads at the same time in the same way as a generator, but it will not generally be so satisfactory to do this. For example, if the motor is connected to a railway gen-

erator, then when there is a weak field current in the synchronous motor an increase of load on the D. C. machine may cause the motor to drop out of step and open its circuit breaker. Thus automatic voltage regulators must be applied with more caution to synchronous motor generator sets than to synchronous motors running idle. From the above consideration, and from the fact that synchronous motors are usually a small proportion of the load of a system, it is probable that the bulk of the reactive Kva. required would be supplied by generators scattered throughout the net-work, and by synchronous phase modifiers running without mechanical load.

The cheapening of the design of water-power plants where there is a large power net-work, and the use of scattered generators for voltage control, tend to make smaller water-powers profitable, and to increase the number of water-powers which can be developed. In the same way, a net-work makes it posssible to supply small communities and isolated customers with electric energy which they could not otherwise obtain.

The growth of high-tension net-works is dependent on the distance separating the power plants and on the economical distance of transmission. If there were a water-power plant every 100 miles throughout the country, it would not be difficult to connect them all into a single net-work, and its operation would probably be safe, judging by some of the large power net-works which already stretch over several hundred miles of territory. But if the power plants were 400 miles apart, and energy could be economically transmitted only 150 miles, the stations could not be interconnected. Now if

some arrangement could be made, such as utilizing the economies of the constant-voltage system, whereby transmission would be made profitable at 300 miles instead of 150, then these stations could be connected to form one net-work. At present, a high-tension net-work covering a section of country often requires the development of very small water-powers, so as to have sources of energy supply at close enough intervals. But with a constant-voltage net-work, with synchronous motors holding the voltage steady at every substation, the distance between generating stations may be much greater, thus permitting the connection of existing transmission-line systems, without the delay and risk of building small intermediate power stations which might be liable to be unprofitable, even with the advantages of operation in a net-work described above.

Thus the principles of constant-voltage transmission can influence very strongly the rate of growth and the profitableness of large transmission line net-works, and may make it immediately commercially practicable to connect any number of power systems into one large net-work of transcontinental extent.

The possible ultimate size of a high-tension net-work will probably not be limited by the danger of having practically unlimited generating capacity connected to one electrical system when a short circuit occurs. The reason is that a short circuit in a net-work extending some hundreds of miles, as the largest modern net-works do, cannot seriously affect distant portions of the net-work, owing to the impedance of the intervening lines. Thus when a short circuit occurs the voltage gradient along the lines is steep, and current is supplied to the short

circuit practically only from the neighboring generating stations.

For example, if a short circuit occurs one hundred miles from a generating station, and the station voltage is held up at 100,000 volts, 60 cycles, a sustained current corresponding only to about 100,000 Kva. will be delivered from the station. This is a small amount compared with that which modern circuit breakers can handle, especially reactance type breakers, which break the current in two steps, or more if necessary. Protective reactance at the station will greatly reduce the above current, so that it may be said that a station one hundred miles away from a short circuit cannot send to it a dangerous amount of current. It is, therefore, evident that the danger from a short circuit is not proportionate to the total power of the net-work, since, when the generators are widely scattered, they are harmless, but the danger is greatest where there are concentrated groups of large ratings in generators. When it is remembered that the 1,500-mile net-work around San Francisco, stretching over 200 or 300 miles of territory, has a power load of only about 100,000 Kw., while practically concentrated loads of from 100,000 to 150,000 Kw. have been successfully handled at New York, Chicago, and Niagara Falls, it does not seem that the handling of short-circuit currents on an extensive net-work is the most difficult feature of electric-power engineering.

Stretches of country a few hundred miles across, devoid of any water-power, have seemed an obstacle which a high-tension net-work could not cross. But the principles of constant-voltage transmission allow the net-work to extend economically across such stretches of

country possibly four or five hundred miles wide. If we assume that the 240-mile line to Los Angeles is profitable, and that a similar line might have been built in the opposite direction, then that city might have been in the centre of a district 480 miles across, without power of its own, and yet receiving power from one large unified net-work. There is, of course, also no reason why steam-power plants should not be connected to such a net-work as well as water-power plants.

# CHAPTER IX

## SYNCHRONOUS MOTORS IN LOCAL DISTRIBUTION

Synchronous motors are most valuable for controlling the voltage of long transmission lines. For this purpose, as has been shown, it is economical to purchase large synchronous machines and run them without mechanical load, merely for the sake of their reactive current. Where the distances of transmission are shorter, the economies produced by synchronous motors are not so great, but it is often possible to utilize the controlling features of motors which are driving mechanical loads. This is done by giving a bonus to energy consumers who install synchronous motors and operate them correctly. The bonus can be made automatic by the installation of certain instruments and the insertion of power-factor clauses in the power contracts.

With long transmission lines, voltage control is of great value largely because there are limitations to the use of high voltage and, as has been shown, voltage variation is the chief difficulty when the voltage is low in proportion to the distance of transmission. With short distances, other difficulties become prominent, and among these are the disadvantages of low power-factor due to the wide-spread use of induction motors.

In the early days of the central station industry, the loads supplied were composed almost entirely of incandescent lamps, and were of practically 100 per cent

power-factor. The disadvantages of low power-factor were exhibited in a striking manner when induction motors became common and the load power-factor was reduced. The capacity of all the electrical apparatus for delivering a certain power load, on which the income depended, was reduced in proportion to the power-factor. This applied to the generators, exciters, transformers, and line conductors. It was also found that the prime movers, such as steam-engines or water-wheels, were out of proportion to the electrical machinery when the power-factor was low. Thus it was impossible to load the prime movers to their full capacity without overloading the electrical apparatus. Therefore, in small power systems, sending power only to short distances, synchronous motors are useful more as power-factor correctors than as voltage regulators.

The simplest method of charging for electric energy, namely, merely on a Kw.-hour rate, is liable to result in low power-factor loads. The natural tendency where there is no contract obligation to maintain a high power-factor is to install induction motors of larger sizes than are necessary, whereas by taking pains in measuring the loads, the motors could be loaded more fully with safety, and a higher power-factor would result. When direct current is required, it will probably be found that an induction motor-generator set or a synchronous converter will be less expensive than a synchronous motor-generator set, though the latter can give the most power-factor correction.

A common form of contract for the sale of electric energy is one which gives a bonus to the customer when the power-factor under load conditions is above, say,

85 per cent. A customer can probably meet this condition by keeping the induction motors approximately fully loaded. Accordingly, when a supply of direct current is needed, there will be little incentive to install a synchronous motor-generator set. The amount of power-factor correction, though it is small, furnished by a synchronous converter, may determine the choice of that type of apparatus. Other factors influencing the choice will be the higher efficiency and lower first cost of a synchronous converter compared with a motor-generator set, and the fact that first-class operation is now obtained with converters by the use of commutating poles. Synchronous converters are best operated with a fixed rheostat setting and are not suitable for giving power-factor control which would necessitate changes in their excitation.

Another form of contract of frequent occurrence is one which gives a bonus increasing in proportion as the power-factor is raised until 100 per cent is reached. This will tend to increase the purchase of synchronous motors. It has been proposed at times to give this bonus by charging for the total Kva.-hours instead of the Kw.-hours, but this is hardly to be recommended, owing to the inadvisability of using a type of meter not now commonly manufactured, and to the doubt whether even good commercial accuracy of metering could be obtained. The bonus might be given according to the minimum readings of a graphic power-factor meter, but it is not always easy to set the rate definitely and precisely by this means. Perhaps the fairest and most accurate way of avoiding the disadvantages of low power-factor is to meter the lagging reactive Kva.-hours and

to charge for them separately and at a rate of, say, one-fourth of the rate for the Kw.-hours.

The metering of the reactive Kva.-hours is accurately done by using a special, simple connection of a standard 3-phase watt-hour meter, as shown in Fig. 8.* On comparing it with the regular connection for energy measurement, Fig. 7, it is seen that the only differences are that in Fig. 8 the connections to the shunt coils of the two elements are interchanged and the connection to one of the series coils is reversed. In Fig. 7 the current

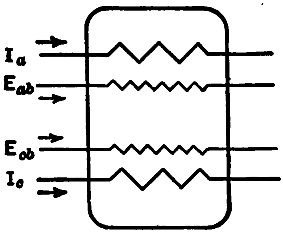


FIG. 7.—Energy Measurement.

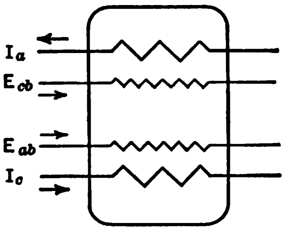


FIG. 8.—Reactive Energy Measurement.

and voltage of each element are 30° out of phase at unity power-factor, and in Fig. 8 they are 90° out of phase, and so register zero at unity power-factor. The readings of the meter in Fig. 8 are multiplied by a constant, $\frac{\sqrt{3}}{2}$, to obtain the reactive Kva.-hours, when the same scale and transformers are used for the two meters shown.

A bonus for high power-factor, such as is given by

* "Watt-hour Meters," by C. W. Baker, *The Canadian Electrical News*, July, 1912.

charging separately for the reactive Kva.-hours, or using the same measurement to calculate the average power-factor, is advisable for cases where the energy-supply company wishes the customer's load to have a leading or a high power-factor at all times of the day, as in the case where the customer is located close to the generating station. This bonus is also applicable where no synchronous motors are installed or are likely to be installed, and where, therefore, power-factor adjustment is not possible. The disadvantage of the above bonus is that it discourages any adjustment of the power-factor, and in many instances it is desirable at times for the sake of the voltage to lower the power-factor by reducing the excitation of the synchronous motors. Some other method than the above of giving the bonus must, therefore, be looked for in a great many cases.

Where the distance to which the energy is sent is sufficient to make the voltage variation due to line drop noticeable, it is very desirable, as mentioned above, to adjust the synchronous motors continually so as to hold the voltage constant. Since this would require them to be operated lagging part of the time, a bonus for very high power-factor such as has been just described would clearly be inadvisable and would be against the real interests of both customer and supply company. This applies to many cases of the distribution of energy from steam plants in large cities, and to all cases where energy is delivered over transmission lines. It may be said that in the majority of cases adjustable power-factor is of more value than high power-factor, and synchronous motors are more valuable as voltage regulators than as power-factor correctors, especially be-

cause, when they are operated as voltage regulators, they also furnish good service as power-factor correctors.

Where voltage variation is troublesome, the supply company wishes synchronous motors to be installed and adjusted for constant voltage, since this reduces the investment in power lines and feeder regulators. This result could be obtained by paying a definite amount as a bonus for each Kva. of synchronous motors available for power-factor control. With isolated customers this would be all that was necessary, since if they did not adjust their voltage properly, they would be practically the only sufferers. In general, however, it would be necessary to impose penalties for poor voltage adjustment, as shown by a graphic voltmeter, since poor adjustment by one customer would be detrimental to the service of neighboring customers, and to the operation of the entire transmission system.

A common method of insuring that synchronous motors are always correctly adjusted for constant voltage is to equip them with automatic voltage regulators, which operate in the same way as with A. C. generators. Although some care must be taken not to reduce the excitation of the motors enough to make them unstable, this method is frequently used and gives successful operation.

In conclusion, the most advisable power-tactor clause for power contracts under average conditions is merely to give a bonus for the installation of synchronous motors in proportion to the reactive Kva. which they can furnish, and to impose penalties when a graphic voltmeter shows that they are not adjusted for constant voltage.

## SUMMARY OF CHAPTER IX

*Synchronous Motors in Local Distribution*

1. A charge for energy at a simple rate per Kw.-hour results in a low P. F. load.

2. A bonus for 85 per cent P. F. encourages a customer to keep induction motors fully loaded, but does not induce him to install synchronous motors.

3. A bonus up to 100 per cent P. F. encourages the installation of synchronous motors, but penalizes voltage control.

4. Where the customer's aid is desired in making use of the constant-voltage method, a bonus should be based on the available Kva. rating of synchronous motors, with penalties when they are not adjusted to give constant voltage.

# CHAPTER X

## COST COMPARISONS

In order to show the possibilities of money-saving in power-factor control, estimated first costs are given for several examples. These comparisons are meant to bring out the general fact that for very long lines the saving may amount to a large proportion of the cost of the line, while for moderate distances the saving is much less, except where the voltage used is unusually low. It is, of course, impossible to assign unit costs which will be precisely equal to those encountered in any given part of the country. However, even if the costs given differ widely from those which ought to be used in a certain case, still they indicate under what conditions the constant-voltage system is worth while merely for reducing line costs, and where it is necessary to depend also on other advantages, and to make use of synchronous motors which also do other work than adjust the power-factor.

The general method which has been used for making the comparison of costs will be found the most suitable for making a first estimate. The largest item which has been omitted is the cost of land. This must be estimated for each separate case, since the cost of land varies extremely in different localities. It has been stated in an authoritative paper on transmission-line construction *

* "Design of Transmission Lines," by Julian C. Smith, Trans. Can. Soc. C. E., October, 1912.

that the right-of-way may vary in cost from $25 per acre for barren land to $250 per acre for good farm land and to $2,000 or $3,000 per acre for land near a large city, and that the total cost of the right-of-way is a large item and may be as high as 25 to 40 per cent of the total cost of the construction. It may be mentioned that the usual practice is to purchase a complete strip of land for the transmission line, and not to purchase rights for merely the land on which the towers rest, as has been done in some instances. Since the constant-voltage system uses fewer lines for a given block of power, and often at lower voltage, it requires much less land, and accordingly its economies will show very much larger when the cost of land is included in the estimate.

Another item which is not directly represented in the comparisons of first costs is that of maintenance and attendance. This item is fairly omitted in a first estimate, because it occurs on both sides of the comparison. Thus, suppose that the limit has been reached for the load of a line, and it is to be decided whether to build a duplicate line, possibly along a new route, or to install synchronous motors to double the rating of the old line. In either case, the running expenses would be increased, for the new line would require repair men and patrols, and the machines would require attendants. Therefore, except where precise results are being figured for a particular case, it is not necessary to estimate the difference in running expenses of the two methods nor the difference in depreciation.

Since one method involves greater power losses than the other, the best way to account for this in the comparisons is to estimate the first cost of the power-plant

equipment needed to generate power for the extra losses, and this has been done at the figure of $100 per Kw. It is obviously incorrect to capitalize the income which might have been obtained from the sale of this extra power, since it is generated in bulk, usually not near a city, and is sent into the transmission lines and lost there. As is well known, the greater part of the cost of power sold to a customer, especially from a water-power plant, is in its distribution and regulation.

The saving due to the constant-voltage system for the 60-mile line, as tabulated, is not large, and at first sight it might appear that for a distance of 60 miles or less synchronous motors are not worth installing in connection with a transmission line. Such, however, might be a quite erroneous conclusion. Before making a final decision, the other items should be taken into account, namely, the saving in land, the possible use of lower voltage involving saving in insulators, transformers, and circuit breakers, the operating advantage of controlling the voltage at the place where the power is used instead of at some distant point by means of long-distance telephone, and, finally, the advantage of having constant voltage at all points so that power can be sold under the best conditions at generating, receiving, and intermediate stations. When all these advantages are considered, there are few cases where synchronous motors cannot be economically used in the design of a transmission system, or where they cannot be profitably applied to a heavily loaded line already constructed.

It is to be noted that the voltage at the receiver end of the constant-voltage lines is always taken at a lower figure than at the generator end, because this gives the

best economy and, it is believed, the best engineering. A line or net-work with not only steady voltage, but equal voltage at all points, may make use of the line construction and insulation more fully, but it will require a very much larger capacity in synchronous phase modifiers, and is, therefore, not so advisable, unless there are special conditions such as the transmission of energy at times in one direction along the line, and at times in the opposite direction.

In the comparisons, where one 3-phase circuit is mentioned, the comparison is made on a unit basis and it is not lost sight of that, for an important transmission project, it is inadvisable to use only one circuit. An exception would be in the case of a net-work, where in case of emergency the power could be transmitted by a roundabout route through the net-work, or the necessary power could be sent from some other source. It is for this reason that net-works can be built of heavy conductor, single circuit lines, which are especially economical when operated by the constant-voltage method.

## I. COMPARISON OF COSTS FOR 241-MILE LINES

This example is partly based on the characteristics of the transmission project from Big Creek to Los Angeles, California.

Cables, 0.95 inch diameter, composed of 596,000 circ. mil of aluminum and a $\frac{7}{16}$-inch diameter steel core. Spacing, 17.5 feet, in a horizontal plane. Frequency, 50 cycles.

| | Varying-Voltage Control | Constant-Voltage Control |
|---|---|---|
| Power delivered | 76,000 Kw. | 76,000 Kw. |
| Power-factor of load | 85% | 85% |
| Number of 3-phase circuits | 4 | 2 |
| Maximum working voltage | 150,000 | 150,000 |
| Voltage at receiver end | 138,000 | 130,000 |
| Reactive drop in transformers at each end | 6% | 6% |
| Reactive drop in protective coils at each end | None | 6% |
| Synchronous phase modifiers at full load | None | 45,000 Kva. |
| Synchronous phase modifiers at no load | None | 38,000 Kva. |
| Voltage variation at supply end | 22% of F. L. value = 27% of N. L. value | None |
| Voltage variation due to line alone | 15% of F. L. value | ..... |
| Efficiency of line alone | 96% | 92% |
| Power-factor at generators | 96% leading | 99.8% lagging |
| APPROXIMATE COSTS | | |
| Towers, ground cables, insulators, and erection | $2,620,000 | $1,310,000 |
| Conductors (aluminum, 25 cts. per lb.) | 2,240,000 | 1,120,000 |
| Synchronous phase modifiers and appurtenances at $10 per Kva. | ......... | 450,000 |
| Extra losses in lines and phase modifiers at $100 per Kw. | ......... | 550,000 |
| Cost of line, exclusive of land | $4,860,000 | $3,430,000 |
| Saving in first cost due to the constant-voltage system | ......... | $1,430,000 |

## II. COMPARISON OF COSTS FOR 120-MILE LINES

| | Varying-Voltage Control | Constant-Voltage Control |
|---|---|---|
| Power delivered | 32,100 Kw. | 32,100 Kw. |
| Power-factor of load | 80% | 80% |
| Number of 3-phase circuits | 3 | 1 |
| Conductors, copper cable | No. 0000 | 300,000 c. m. |
| Spacing, effective | 14 ft. | 14 ft. |
| Frequency | 60 cycles | 60 cycles |
| Maximum working voltage | 110,000 | 110,000 |
| Voltage at receiver end | 93,500 | 95,000 |
| Reactive drop in transformers at each end | 6% | 6% |
| Resistance drop in transformers at each end | 0.75% | 0.75% |
| Synchronous phase modifiers at full load | None | 24,100 Kva. |
| Synchronous phase modifiers at no load | None | 13,900 Kva. |
| Voltage variation at supply end | 20% of F. L. value = 25% of N. L. value | None |
| Efficiency of line alone | 95.3% | 92.6% |
| Power-factor at generators | 93% lagging | 97% lagging |
| **APPROXIMATE COSTS** | | |
| Towers, ground cables, insulators, and erection | $738,000 | $252,000 |
| Copper at 18 cts. per lb. | 651,000 | 308,000 |
| Synchronous phase modifiers and ensuing expenses at $10 per Kva. | ......... | 241,000 |
| Extra losses in line and phase modifiers at $100 per Kw. | ......... | 208,000 |
| Cost of line, exclusive of land | $1,389,000 | $1,009,000 |
| Saving due to the constant-voltage system | ......... | $380,000 |

## III. COMPARISON OF COSTS FOR 60-MILE LINES

| | Varying-Voltage Control | Constant-Voltage Control |
|---|---|---|
| Power delivered | 19,500 Kw. | 19,500 Kw. |
| Power-factor of load | 80% | 80% |
| Number of 3-phase circuits | 3 | 1 |
| Conductors, copper cable | No. 000 | 250,000 c. m. |
| Spacing, effective | 7 ft. | 7 ft. |
| Frequency | 60 cycles | 60 cycles |
| Maximum working voltage | 60,000 | 60,000 |
| Voltage at receiver end | 50,400 | 50,000 |
| Reactive drop in transformers at each end | 3.5% | 3.5% |
| Resistance drop in transformers at each end | 0.75% | 0.75% |
| Synchronous phase modifiers at full load | None | 13,500 Kva. |
| Synchronous phase modifiers at no load | None | 10,000 Kva. |
| Voltage variation at supply end | 17% of F. L. value = 20% of N. L. value | None |
| Efficiency of line alone | 93% | 90.7% |
| Power-factor at generators | 78.6% lagging | 94.2% lagging |
| **APPROXIMATE COSTS** | | |
| Towers, ground cables, insulators, and erection | $234,000 | $84,000 |
| Copper at 18 cts. per lb. | 258,000 | 128,000 |
| Synchronous phase modifiers and ensuing expenses at $10 per Kva. | ........ | 135,000 |
| Extra losses in line and phase modifiers at $100 per Kw. | ........ | 113,000 |
| Cost of line, exclusive of land | $492,000 | $460,000 |
| Saving due to the constant-voltage system | ........ | 32,000 |

## IV. COMPARISON OF COSTS FOR 4-MILE LINES

| | Varying-Voltage Control | Constant-Voltage Control |
|---|---|---|
| Power delivered | 240 Kw. | 240 Kw. |
| Power-factor of load | 80% | 80% |
| Number of 3-phase circuits | 2 | 1 |
| Conductors, copper wire | No. 00 | No. 00 |
| Spacing, effective | 1 ft. | 1 ft. |
| Frequency | 60 cycles | 60 cycles |
| Maximum working voltage | 2,200 | 2,200 |
| Voltage at receiver end | 2,000 | 2,000 |
| Synchronous phase modifier at full load | None | 198 Kva. |
| Synchronous phase modifier at no load | None | 180 Kva. |
| Voltage variation | 10% | None |
| Efficiency of line alone | 92.6% | 90.7% |
| Power-factor at generators | 78.5% lagging | 99.9% leading |
| APPROXIMATE COSTS | | |
| Copper wire at 18 cts. per lb. | $9,120 | $4,560 |
| Space on pole lines | 2,500 | 1,600 |
| Synchronous phase modifier and ensuing expenses at $15 per Kva. | ...... | 2,970 |
| Extra losses in line and phase modifier at $100 per Kw. | ...... | 1,880 |
| Reduction in generator load at $10 per Kva | 650 | ...... |
| | $12,270 | $11,010 |
| Saving due to constant-voltage system | ...... | 1,260 |

NOTE: As the above saving in copper could have been made by using transformers for 6,600 or 11,000 volts, the constant-voltage system would probably not be economical unless the voltage is limited in some way to 2,200 volts, or unless contracts can be made to obtain the required voltage control from the customers' synchronous motors. See Chap. IX.

## CHAPTER XI

### FUTILITY OF USING UNUSUALLY LARGE VOLTAGE VARIATION

The constant-voltage method enables a transmission line to deliver a load which would produce a voltage regulation of 40 or 50 per cent, were it not for the continual adjustments in power-factor made in order to hold the voltage constant. Ordinarily, a load which gives a voltage variation of 20 or 25 per cent is all that can be taken care of either by hand regulation or by automatic regulators, since both the momentary variations and the total variations become troublesome. Some water-power plants, however, deliver all their power over a transmission line to a city, no power being sold close to the power plant, and so the total voltage variation at the generators is of small moment.

The question, therefore, may well arise, "If an automatic regulator could be developed which would be able to produce satisfactorily steady voltage at the city even when the total variation at the generators is 50 per cent, would it give the advantages of the constant-voltage system without the expense of synchronous motors?" The answer is that when a varying-voltage line delivers so much power as to give 40 or 50 per cent regulation, the large lagging current sent over the line from the generators reduces the generator power-factor and the transmission-line efficiency to a remarkable degree, so much so that synchronous motors are worth their price merely for reducing the Kva. load and the Kw. load on the generating station, without considering

the advantages of a more constant generator voltage. This is best shown by the attached example, where the saving in power, allowing fully for the losses in the synchronous phase modifiers, produces a large net saving for the constant-voltage system, at an equal power load.

It is seen from this example that unless synchronous motors are thought to introduce an element of complication and unreliability into a transmission system, there would be no advantage gained by producing a voltage regulator which could operate with larger variations in generator voltage than those used at present.

This example deals with a line of medium length, which would not have a troublesome amount of charging current, and yet synchronous phase modifiers are seen to be economical. It is sometimes stated that synchronous phase modifiers are used with long transmission lines because of difficulties caused by charging current, but this statement is not correct, even where the charging current produces a rise at no load equal to 10 or 15 per cent of normal voltage. Phase modifiers are to be used with long lines chiefly because they save dollars and cents, largely by saving line costs, and also, as in this chapter, by increasing the efficiency of transmission and the power-factor at the generators. The savings are greater with very long lines because line costs are then more prominent, and not because the charging current is greater. In fact, for a given line, the lower the voltage, the less will be the charging current, and yet the greater will be the financial necessity for phase modifiers. It will be shown in Chapter XIII that the difficulties due to the charging current are probably not equal to the advantages which it gives.

### COMPARISON OF THE OPERATION OF A LINE AT A GIVEN HEAVY LOAD, WITH AND WITHOUT SYNCHRONOUS MOTORS

Let the line be the same as the varying-voltage line of Example III, Chapter X. Length, 60 miles; one 3-phase circuit; frequency, 60 cycles; conductor, No. 000 copper cable; spacing, 7 feet effective; reactive drop in transformers at each end, 5 per cent.

| | Varying-Voltage Control | Constant-Voltage Control |
|---|---|---|
| Power delivered | 9,500 Kw. | 9,500 Kw. |
| Power-factor of load | 80% | 80% |
| Maximum working voltage | 60,000 | 60,000 |
| Synchronous phase modifiers at no load and full load | None | 5,900 Kva. |
| Voltage variation at supply end | 30% of F. L. value = 43% of N. L. value | None |
| Efficiency of transmission | 84.5% | 92.8% |
| Generator Kw | 11,250 Kw. | 10,250 Kw. |
| Power-factor at generators | 70% lagging | 97.5% lagging |
| Generator Kva | 16,000 Kva. | 10,500 Kva. |
| APPROXIMATE FIRST COSTS | | |
| Net saving in power at $100 per Kw | $70,000 | ...... |
| Saving in generator rating at $10 per Kva | 55,000 | ...... |
| Cost of synchronous motors at $10 per Kva | ...... | $59,000 |
| Approximate saving from synchronous motors | ...... | 66,000 |

## CHAPTER XII

### WORKING METHODS OF CALCULATION

A BETTER argument for the constant-voltage method of control than general statements or examples of typical or actual transmission lines is to estimate for the line in which one is most interested, the benefit to be obtained from installing synchronous motors. This estimation cannot be properly made by merely using the calculations which are usual for varying-voltage lines. Thus, it is not consistent to calculate the regulation of the line between full load and no load, since the power-factor is not a fixed quantity. So also, the benefits from the synchronous motors are not best shown by calculating the amount by which they can raise the power-factor of the load, since the benefit given to the transmission system by the synchronous motors by controlling the voltage is generally of much greater value than the benefit given by raising the power-factor. Although it is possible to operate a transmission line with a small number of synchronous motors and with voltage variation, probably the best results will be obtained by installing enough synchronous motors so that they can be depended on to hold the voltage constant at all parts of the line for the range of load which the line will normally carry. The working methods of calculation which will be described will be based on this latter condition.

Working methods of calculation should, if possible, conform to the following conditions: The results to be

calculated should be selected because they are of the greatest engineering value, and not because they are the most convenient to obtain. The results should not be obtained by trial and error, but by a direct process of calculation. Graphical methods should be supplemented by mathematical methods for obtaining the same results with greater accuracy. In the present case it may also be noted that the results are obtained without using tables of trigonometric functions of angles nor tables of hyperbolic functions; however, tables of transmission-line constants will be found convenient.

With comparatively short lines, up to twenty or thirty miles in length, the electrostatic capacity of the

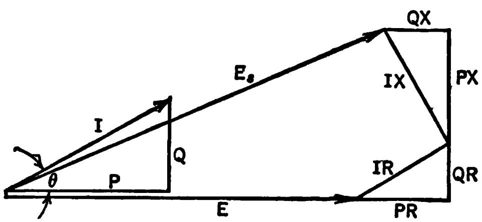


FIG. 9.—Vector Diagram of A. C. Line with Leading Current, Conditions Given at Receiver End.

line may be neglected, and this greatly simplifies the electrical formulas. The fundamental relationships of the various electrical quantities connected with an A. C. line, neglecting the electrostatic capacity, are completely shown by the vector diagram, Fig. 9, when the load conditions are specified at the receiver end of the line.

This diagram is drawn for a leading current since, with the formulas being considered, a leading reactive current is taken as a positive quantity, and a lagging one

as a negative quantity. The vector diagram for a lagging current is the same as Fig. 9, except that $Q$ is always drawn in the opposite direction. It is shown in Fig. 4, Chapter IV. The quantities shown in Fig. 9 are the same as those in Fig. 4, except that $Q$ represents a leading current instead of lagging.

The difference between the voltages $E_s$ and $E$ is equal to the voltage drop in the conductors, and is made up of the resistance drop, $I\,R$, in phase with $I$, and the reactance drop, $I\,X$, in quadrature with $I$. Using components which are at right angles to one another, we obtain the relationship

$$E_s^2 = (E + P\,R - Q\,X)^2 + (P\,X + Q\,R)^2 \quad . \quad (1)$$

In a constant-voltage line, $E$ and $E_s$ are constant, and $P$ and $Q$ vary as the load changes. The above equation, therefore, reduces to the following equation between $P$ and $Q$:

$$P^2 + Q^2 + \frac{2\,E\,R}{R^2 + X^2}P - \frac{2\,E\,X}{R^2 + X^2}Q - \frac{E_s^2 - E^2}{R^2 + X^2} = 0 \quad (2)$$

This is seen to be the equation of a circle. This circle is easily drawn on cross-section paper, and it gives a satisfactory graphical solution of an important problem in connection with a constant-voltage line, namely, that of finding the ratings in synchronous motors required for various loads, with given terminal voltages. The best units to use are Kw. and Kva. It is recommended that the circle diagram of a constant-voltage line be drawn in all cases, because it is a short operation and because it gives a comprehensive idea of relative magnitudes and of certain limits to the power load of the line. With fine section paper, very close results may be obtained.

For more accurate work, any point on the diagram may be directly calculated by the methods in Tables I–IV.

Instructions for drawing the circle diagram of a constant-voltage line are given in (1), Table I, when the load is specified at the receiver end. An ordinate to the circle represents the reactive Kva. in the line at the receiver end, for an amount of delivered power in kilowatts represented by the corresponding abscissa. An ordinate to the straight line represents the reactive Kva. of the load, and an ordinate to the ellipse shows the reactive Kva. of the synchronous motors. Reactive Kva. as shown on the diagram are leading when the ordinate is positive and lagging when it is negative. See Fig. 10, which shows the circle diagram for the line in Example 1, Chapter X.

The straight line is most easily drawn by plotting an abscissa, $\cos \theta$, and an ordinate, $-\sin \theta$, to give one point on the line. The ordinates of the straight line must be added to those of the circle, because the synchronous motors must correct the lagging Kva. of the load, as well as provide the reactive Kva. in the line at the receiver end.

The theoretical limit of the load of a line may be read directly from the circle diagram, or it may be calculated by formula (2). This theoretical limit is so much larger than the regular load of an ordinary line with rather small conductors, as about No. 000, that it is of little importance in such cases. It is of considerable importance, however, with conductors as large as 350,000 circ. mil copper, since the efficiency is high right up to the theoretical limit. The addition of reactance directly decreases the theoretical limit. In practice, the

effect of the theoretical limit is felt with very heavy lines by a rapid increase in the number of phase modifiers required as this limit begins to be approached, and this imposes an economical limit on the load.

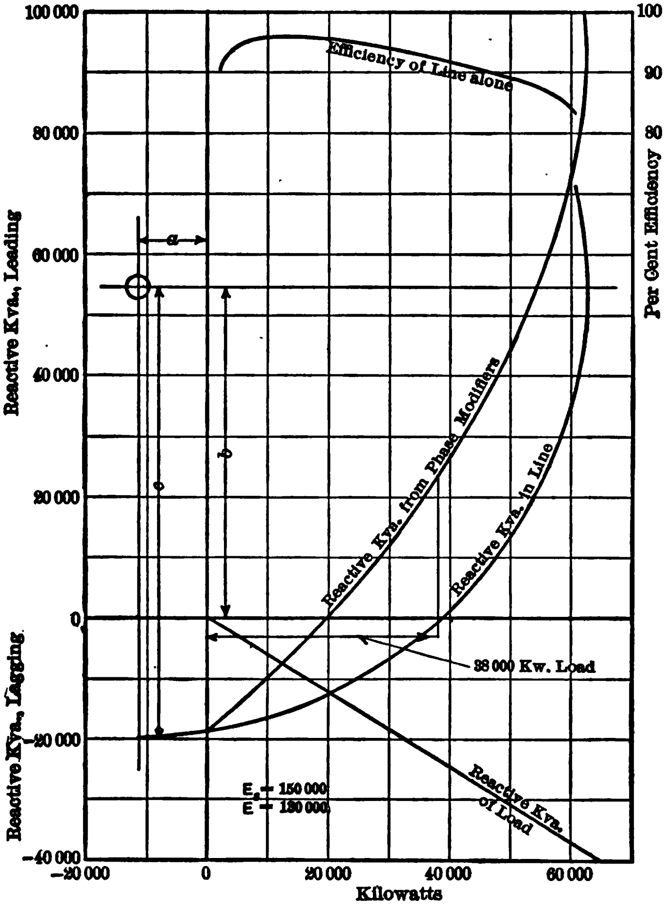


FIG. 10.—Circle Diagram of a Constant-Voltage Transmission Line, Conditions given at Receiver End.

The quadratic equation for obtaining $Q$, given in (3), Table I, is to be used only when more exact results are desired than can be obtained from the circle diagram. This equation is derived directly from equation (1), of this Chapter. While a quadratic equation may appear forbidding at first sight, experience will show that it is less labor to obtain at once a desired result than to obtain it, perhaps only approximately, by a roundabout

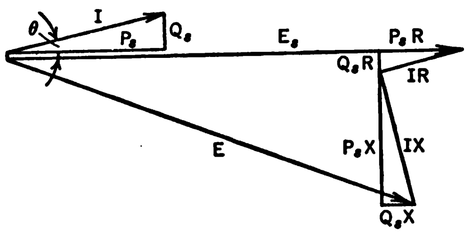


FIG. 11.—Vector Diagram of A. C. Line with Leading Current, Conditions given at Supply End.

method of trial and error. The remaining equations of Table I are obtained in the same way as similar equations for varying-voltage lines.*

The equations of Table II, where conditions are specified at the supply end of the line, are very similar to those of Table I. They depend, however, on a different vector diagram, which is shown in Fig. 11.

The equation for the receiver voltage derived from the vector diagram, Fig. 11, is similar in form to equation (1) of this Chapter, and is

$$E^2 = (E_s - P_s R + Q_s X)^2 + (P_s X + Q_s R)^2 \quad . \quad (3)$$

When the line is long enough to make the effect of

* See "Transmission Line Formulas," D. Van Nostrand Company, 1913.

its electrostatic capacity appreciable, the accurate calculation of the line can be obtained most easily by using convergent series, as in Tables III and IV. By this method the use of hyperbolic functions is avoided.

Since the convergent series require the use of complex numbers, which are distinguished by the use of the letter "$j$," a very short outline of the rules for using these numbers will be given, in order to make the instructions for the tables quite complete.

Each of the complex quantities, $Z = R + j\,X$, $Y = G + j\,B$, $C + j\,D$, etc., is composed of two parts, the first, a "real" term and the second an "imaginary" term, which is multiplied by the letter $j$. In adding complex numbers, the imaginary terms must be kept separate from the others. Thus,

$$6 + j\,2 \text{ added to } 4 + j\,9 = 10 + j\,11$$

In multiplying two complex quantities, one follows the rules of ordinary algebra, $j$ being treated as an algebraical quantity which has the property that $j \times j = j^2 = -1$.

From this it follows that,

$$\begin{aligned} -j \times j &= +1 \\ j^3 &= -j \\ j^4 &= +1 \\ j^5 &= +j, \text{ etc.} \end{aligned}$$

Thus $(6 + j\,2) \times (4 + j\,9)$ is worked out as follows:

$$\begin{array}{r} 6 + j\,2 \\ 4 + j\,9 \\ \hline +24 + j\,8 \\ -18 + j\,54 \\ \hline +\ \ 6 + j\,62 \end{array}$$

In using complex quantities with alternating-current problems, quantities which are in quadrature, that is, which have 90° lead or lag, are called imaginary and are multiplied by $j$. Thus, a quadrature current and a reactance are both multiplied by $j$, and when they are multiplied together in order to obtain the voltage drop, the drop is found to be a negative quantity, that is, it has 180° lag, which agrees with the physical fact that a quadrature current through a reactance has a negative in-phase voltage drop. It is found that the mathematical properties of complex quantities agree completely with the physical characteristics of alternating currents of sine wave form, and that complex quantities furnish the easiest means of calculating the results indicated by vector diagrams.

For calculating the convergent series for transmission lines, $Y$ and $Z$ are first written down as complex numbers. The necessary data can be obtained from tables of resistance, reactance, and capacity susceptance. The leakage conductance is generally omitted from engineering calculations, as it is due to insulator leakage and corona loss, which ought not, for good practice, to be appreciable at the operating voltage. By multiplying the complex quantities $Y$ and $Z$, the product $YZ$ may be obtained as a complex number composed of a single real term and a single $j$ term. From this are obtained

$$\frac{YZ}{2}, \quad \frac{YZ}{4} \quad \text{and} \quad \frac{YZ}{6},$$

and by multiplying again,

$$\frac{Y^2 Z^2}{2 \cdot 3 \cdot 4} \quad \text{and} \quad \frac{Y^2 Z^2}{2 \cdot 3 \cdot 4 \cdot 5}$$

may be found. The series should be continued until the terms are negligible in comparison with $\frac{Y Z}{2}$, which will be evident by inspection while doing the work. It is seldom necessary to include more than the terms in $Y^2 Z^2$ in power transmission work.

By addition of terms obtained above, the values of

$$\frac{Y Z}{2} + \frac{Y^2 Z^2}{2 \cdot 3 \cdot 4} + \text{etc. and } \frac{Y Z}{2 \cdot 3} + \frac{Y^2 Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \text{etc.}$$

are obtained, each as a complex number of two terms.

For obtaining the quantities required in using Table III, multiply $E$ by the value found for $\left(\frac{Y Z}{2} + \text{etc.}\right)$ and add it to $E$, thus obtaining $E' + j E''$. In a similar way, multiply $R + j X$ by $\left(\frac{Y Z}{2 \cdot 3} + \text{etc.}\right)$, and add it to $R + j X$, the result being $R' + j X'$. Similar operations are used for Table IV.

The form of the equations in Tables III and IV is very similar to that in Tables I and II. Methods for deriving most of the formulas are the same as for varying-voltage lines.

The formulas of Tables I to IV are suitable for net-works. Each junction point of a constant-voltage transmission line net-work has a definite voltage, and so each line in the net-work can be calculated, the variable item being the amount of real power transmitted over the line. Given the amount of real power, then the rating of synchronous motors to be placed at the two junction points for the sake of the line between

them, the efficiency, power-factor, etc., can be directly calculated.

Although the term $Y$ for the admittance of the line may include the conductance between wires due to insulator leakage and corona loss, it is not usual to include the effect of leakage and corona when calculating the line characteristics, but to take $Y$ as equal to only the capacity susceptance. The reason for this is that insulator leakage under ordinary conditions is generally negligible compared with other line losses, and corona loss ought to be quite negligible under normal conditions. When corona loss once starts, it increases at a very rapid rate as the voltage goes up. If corona loss is at all appreciable under normal conditions, then the energy loss during unfavorable weather conditions of any kind becomes very serious. Accordingly, it has been found best to operate lines well below their corona point. For this reason Table V has been inserted. This was published in *The General Electric Review* of December, 1912, by Mr. F. W. Peek, Jr., who has made the most authoritative determination of the laws of corona loss of transmission lines. From this table, the voltage may be found at which corona loss begins to be appreciable for any given line under normal conditions; it is generally not desirable to operate above these voltages.

### *Example I*

Find the line efficiency, the generator P. F., and the reactive Kva. required from synchronous phase modifiers, for the four-mile, constant-voltage line, Example IV, Chapter X.

Conditions given:

$$E_s = 2{,}200$$
$$E = 2{,}000$$
$$R = 1.69$$
$$X = 2.16$$
$$P = \frac{1{,}000 \times 240}{2{,}000} = 120 \text{ total amperes.}$$
$$\cos\theta = 0.80$$

Then, from (3), Table I,

$$a_1 = 7.50$$
$$b_1 = 4{,}320$$

and $$c_1 = 14{,}400 \times 7.50 + 4{,}000 \times 120 \times 1.69 + 4{,}000{,}000 - 4{,}840{,}000$$
$$= 79{,}000$$

Then $$Q = \frac{4{,}320 - \sqrt{4{,}320^2 - 7.50 \times 79{,}000}}{7.50}$$
$$= +9.2 \text{ total amperes.}$$

Therefore the reactive Kva. in the line at the receiver end are leading.

The reactive Kva. of synchronous motors are, by (6), Table I,

$$2 \times 9.2 + 2 \times \frac{0.6}{0.8} \times 120 = 198 \text{ Kva.}$$

The power-factor at the generators is, by (10),

$$100 \frac{240{,}000 + 14{,}490 \times 1.69}{2{,}200 \times 120.3} = 99.9\%$$

The reactive Kva. at the generators are, by (11),

$$\frac{2{,}000 \times 9.2 - 14{,}490 \times 2.16}{1{,}000} = -13 \text{ Kva.}$$

Therefore, the generator P. F. is lagging.

The efficiency of the line, by (12), is

$$\frac{100 \times 240{,}000}{240{,}000 + 14{,}490 \times 1.69} = 90.7\%$$

*Example II*

Find the data for drawing the circle diagram for the 241-mile, constant-voltage line, Example I, Chapter X. See Table III.

Conditions given:

$$E = 130{,}000$$
$$E_s = 150{,}000$$

Impedance of transformers and reactance coils to be included.

$$Z = 50.9 + j\,271.5$$
$$\frac{Y}{2} = \qquad + j\;\;0.000535$$

---

$$\frac{Y\,Z}{2} = -\,0.1453 + j\,0.0272$$
$$\frac{Y\,Z}{4} = -\,0.0726 + j\,0.0136$$
$$\frac{Y\,Z}{6} = -\,0.0484 + j\,0.0091$$

---

$$+\,0.0035 - j\,0.0007$$
$$-\,0.0001 - j\,0.0007$$

---

$$\frac{Y^2\,Z^2}{2.3.4} = +\,0.0034 - j\,0.0014$$
$$\frac{Y^2\,Z^2}{2.3.4.5} = +\,0.0007 - j\,0.0003$$

Further terms of the series are evidently negligible. Adding the series,

$$\frac{Y\,Z}{2} + \frac{Y^2\,Z^2}{2.3.4} + \text{etc.} = -\,0.1419 + j\,0.0258$$

$$\text{and } \frac{Y Z}{2.3} + \frac{Y^2 Z^2}{2.3.4.5} + \text{etc.} = -0.0477 + j\,0.0088$$

$$E\left(\frac{Y Z}{2} + \text{etc.}\right) = -18{,}450 + j\,3370$$

$$E = 130{,}000$$

$$E' + j\,E'' = 111{,}550 + j\,3{,}370$$

$$Z\left(\frac{Y Z}{2.3} + \text{etc.}\right) = -4.8 - j\,12.5$$

$$Z = 50.9 + j\,271.5$$

$$R' + j\,X' = 46.1 + j\,259.0$$

$$a' = -130\,\frac{111{,}550 \times 46.1 + 3{,}370 \times 259}{69{,}200}$$

$$= -11{,}300 \text{ Kw.}$$

$$b' = +130\,\frac{111{,}550 \times 259 - 3{,}370 \times 46.1}{69{,}200}$$

$$= +54{,}100 \text{ Kva.}$$

$$c' = +130\,\frac{150{,}000}{263.1}$$

$$= +74{,}100 \text{ Kva.}$$

With these values, the circle diagram can be drawn, as in Fig. 10.

# CHAPTER XIII

## EFFECT OF CHARGING CURRENT

THE electrostatic capacity of a transmission line, which causes the well-known charging current observed at no load, has often been stated to be a considerable disadvantage. However, the reverse is probably the case. A casual inspection of Tables I and III will not show at once the true effect of line capacity; neither can its effect be judged by observation of the line when power is being transmitted. Since the charging current is only observable at no load, when it often necessitates the running of a comparatively large quantity of generators and when it also produces a rise in the receiver voltage, it is sometimes forgotten, or not considered, that the charging current is as helpful in reducing the generator load and raising the power factor at full load as it is troublesome in increasing the work of the generators at no load. Thus the charging current may be said to improve the load factor, since it is helpful at times of heavy load, and the reverse at times of light load. Although the charging current often has a considerable effect on the line voltage, this effect is present at both no load and full load, and so the result on the regulation, that is, the voltage variation, is very slight. What effect does exist results in making the line regulation better. A careful comparison of the formulas, including

and neglecting capacity effect, as in the attached examples of 200-mile lines, shows that it is possible to transmit about 10 per cent more power over a 200-mile line of given maximum voltage, either of varying-voltage or constant-voltage control, than it would be if there were no such thing as line capacity. The efficiency of transmission and the generator power-factor are also improved by the line capacity.*

In connection with one phenomenon of transmission-line operation, the charging current may be considered to be a disadvantage. This occurs when the load is suddenly disconnected, as when the receiver station circuit-breakers open, and the excitation of the generators is, for the time being, unaltered. If there is not a very large number of generators connected, the charging current will form a heavy leading power-factor load for them. This will help magnetize their fields, so that the sustained line voltage may be as high as twice the normal voltage, as indicated in the attached examples. This might be more harmful, if not prevented, than the transient voltages which also occur on the opening of circuit-breakers. The above phenomenon, however, has not proved to be a very troublesome feature of operation. It can generally be guarded against sufficiently by keeping enough generators on the line to carry the charging current easily and by keeping the generator voltage closely regulated. Although the percentage regulation of a generator between a lagging load and full load of leading power-factor current is large, being much

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* "Maximum Loads of Transmission Lines," *Electric Journal*, September, 1913.

more than the regulation from full load to no load, it does not seem necessary to add the generator regulation with unchanged field current to the regulation of the line and transformers, in order to obtain the maximum voltage variation for successful practical operation.

### Effect of Charging Current in a 200-Mile Line

Conductors, copper cable, 250,000 circular mils.
Spacing of conductors, 10 feet, flat spacing.
Effective spacing $\sqrt[3]{10 \times 10 \times 20} = 12.6$ feet.
Frequency, 60 cycles.
Resistance of line per conductor, 45.7 ohms.
Reactance of line per conductor, 160 ohms.
Capacity susceptance of line per conductor, 0.00107 ohms.
Resistance of transformers and protective coils at each end, referred to high tension, 4.1 ohms.
Reactance of transformers and protective coils at each end, referred to high tension, 64.5 ohms.
Maximum working voltage, 115,000 volts.
Power-factor of load, lagging, 85%.

### Varying-Voltage Lines, No Synchronous Phase Modifiers Used

| | Line 200 Miles in Length, with Transformers and Protective Coils at Each End. | Short Line of Same Electrical Characteristics, but Without Capacity. |
|---|---|---|
| Voltage variation at generator end | 92,000 to 115,000 | 92,000 to 115,000 |
| Steady voltage at receiver end. | 108,000 | 92,000 |
| Kilowatts delivered | 10,100 Kw. | 8,200 Kw. |
| Efficiency of transmission at full load | 95% | 93% |
| Power-factor at generators, lagging | 89% | 73% |
| Generator Kva. at no load with normal voltage at receiver | 10,400 Kva. | ........... |
| Approximate voltage at receiver end when the receiver circuit-breakers open, enough generators being connected to carry the charging current. | 200,000 volts | 155,000 volts |

CONSTANT-VOLTAGE LINES, SYNCHRONOUS PHASE MODIFIERS USED

| | Long Line | Short Line |
|---|---|---|
| Constant voltage at generator end. | 115,000 volts | 115,000 volts |
| Constant voltage at receiver end.. | 81,000 volts | 81,000 volts |
| Efficiency of transmission at full load.......................... | 85% | 85% |
| Kilowatts delivered.............. | 23,100 Kw. | 21,100 Kw. |
| Synchronous phase modifiers required at full load............. | 13,900 Kva. | 15,800 Kva. |
| Synchronous phase modifiers required at no load.............. | 13,900 Kva. | 9,500 Kva. |
| Generator Kva. at full load...... | 29,600 Kva. | 30,300 Kva. |
| Power-factor at generators, lagging | 92% | 82% |
| Theoretical limit of load......... | 29,600 Kw. | 27,600 Kw. |
| Generator Kva. at no load and at normal voltage................ | 7,100 Kva | 13,500 Kva. |
| Approximate voltage at receiver end when the receiver circuit-breakers open, enough generators being connected to carry the charging current.............. | 200,000 volts | 150,000 volts |

Although without any reference to the subject-matter of this chapter, the following approximate first costs of the above lines have been inserted, as they form an addition to the cost comparisons of Chapter X.

| | |
|---|---|
| Cost of two varying-voltage 200-mile lines: | |
| Copper cables, at 18 cts. per lb...................... | $856,000 |
| Towers, insulators, ground cables, and erection....... | 896,000 |
| | $1,752,000 |
| Cost of one constant-voltage 200-mile line: | |
| Copper cables, at 18 cts. per lb...................... | $428,000 |
| Towers, insulators, ground cables, and erection....... | 448,000 |
| Synchronous phase modifiers, and ensuing expenses, at $10 per Kva.................................. | 139,000 |
| Extra losses, at $100 per Kw......................... | 272,000 |
| | $1,287,000 |
| Saving due to the constant-voltage method, omitting cost of land.................................. | $465,000 |

## TABLE I

### *Formulas for Short Constant-Voltage Lines*
### *Conditions Given at Receiver End*

These formulas are exact when the line is short. When the line is 20 miles long, they are correct within approximately $^{1}/_{10}$ of 1 per cent of line voltage.

*Conditions Given:*

$E$ = Voltage at receiver end.
$E_s$ = Voltage at supply end.
$R$ = Resistance of one conductor.
$X$ = Reactance of one conductor.
(See Tables VII–XII, "Transmission Line Formulas")
$P$ = In-phase current of the load, in total amperes.
$\cos \Theta$ = Power-factor, lagging, of the load.

(1) *Circle Diagram.*

Describe a circle with centre $(a, b)$ and radius $c$, where

$$a = -\frac{1}{1{,}000}\frac{E^2 R}{R^2 + X^2}$$

$$b = +\frac{1}{1{,}000}\frac{E^2 X}{R^2 + X^2}$$

$$\text{and } c = +\frac{1}{1{,}000}\frac{E\,E_s}{\sqrt{R^2 + X^2}}$$

Draw a straight line at an angle $\theta$ below the base line, where $\cos \theta$ is the power-factor, lagging, of the load.

By means of a pair of dividers, add the ordinates of the straight line to those of the circle, thus plotting the ellipse giving the Kva. of synchronous motors required.

(2) *Theoretical Limit of Load, in Kilowatts.*

The maximum load is

$$c + a$$

$$\text{or } \frac{1}{1{,}000}\left(\frac{E\,E_s}{\sqrt{R^2 + X^2}} - \frac{E^2 R}{R^2 + X^2}\right)$$

This is numerically less than $c$, since $a$ is negative. It may be read from the circle diagram, as it is the farthest distance to the right reached by the circle or the ellipse.

(3) *Reactive Kva. in the Line at Receiver End.*

For a more precise value than that obtained from the circle diagram, solve the quadratic equation:

$$a_1 Q^2 - 2\, b_1 Q + c_1 = 0$$

where $a_1 = R^2 + X^2$

$b_1 = E\,X$

and $c_1 = P^2 (R^2 + X^2) + 2\, E\,P\,R + E^2 - E_s^2$

Then $Q = \dfrac{b_1 - \sqrt{b_1^2 - a_1 c_1}}{a_1}$

Use only the negative value of the radical. $Q$ is the reactive current, and $\dfrac{E\ Q}{1{,}000}$, the reactive Kva., in the line at the receiver for a given Kw. load $\dfrac{E\ P}{1{,}000}$. When $Q$, as found above, is positive, it represents leading reactive Kva. in the line at the receiver, and lagging, when it is negative.

(4) *Line Power-factor at Receiver End.*

$$\frac{100\,P}{\sqrt{P^2 + Q^2}} \text{ per cent.}$$

$P$ is determined from the Kw. load, and $Q$ is either read from the circle diagram or calculated by means of (3)

(5) *Reactive Kva. of Load.*

$$\frac{E\,Q_l}{1{,}000} = -\frac{E\,m\,P}{1{,}000} \text{ (a negative quantity)}$$

where $Q_l$ is the reactive component of the load current, $\frac{E\,P}{1{,}000}$ is the load in kilowatts at a lagging P. F. $\cos\theta$, and $m$ is a constant equal to $\frac{\sin\theta}{\cos\theta}$.

(6) *Reactive Kva. of Synchronous Motors.*

This may be read from the circle diagram, or it may be more accurately derived by using the calculated value of $Q$ in the expression

$$\frac{E\,Q}{1{,}000} + \frac{E\,m\,P}{1{,}000} \text{ Kva.}$$

When the above expression is positive, the current in the synchronous motors is leading, and they require strong field current. When the expression is negative the current is lagging, and the synchronous motors operate with weakened fields.

(7) *Line Losses, in Kilowatts.*

$$\frac{(P^2 + Q^2)\,R}{1{,}000}$$

$P$ is determined from the Kw. load, and $Q$ is either read from the circle diagram or calculated by means of (3).

(8) *Kw. at Generators.*

$$\frac{E\,P + (P^2 + Q^2)\,R}{1{,}000}$$

(9) *Kva. at Generators.*

$$\frac{E_s\,\sqrt{P^2 + Q^2}}{1{,}000}$$

(10) *Power-factor at Generators.*

$$100\,\frac{E\,P + (P^2 + Q^2)\,R}{E_s\,\sqrt{P^2 + Q^2}} \text{ per cent.}$$

(11) *Reactive Kva. at Generators.*

$$\frac{E\,Q - (P^2 + Q^2)\,X}{1{,}000}$$

When this quantity is positive, the reactive Kva. and the generator P. F. are leading, and when it is negative, they are lagging.

(12) *Efficiency of the Transmission Line.*

$$\frac{100\,E\,P}{E\,P + (P^2 + Q^2)\,R} \text{ per cent.}$$

(13) *Amperes per Wire.*

As all the expressions for current are in total amperes, it may be noted that

$$\text{amperes per wire, three-phase,} = \frac{\text{total amperes}}{\sqrt{3}}$$

$$\text{amperes per wire, two-phase,} = \frac{\text{total amperes}}{2}$$

and amperes per wire, single-phase = total amperes. For single-phase lines use $2\,R$ and $2\,X$ in place of $R$ and $X$.

(14) *Allowance for Extra Impedance.*

The resistance and reactance of the transformers and protective coils, referred to high tension, may be added to the resistance and reactance of the line. This involves no error or approximation at all when the electrostatic capacity of the line is not to be considered.

## TABLE II

*Formulas for Short Constant-Voltage Lines*
*Conditions Given at Supply End*

These formulas are exact when the line is short. When the line is 20 miles long, they are correct within approximately $^1/_{10}$ of 1 per cent of line voltage.

*Conditions Given:*

$E$ = Voltage at receiver end.
$E_s$ = Voltage at supply end.
$R$ = Resistance of one conductor.
$X$ = Reactance of one conductor.
(See Tables VII–XII, "Transmission Line Formulas.")
$P_s$ = Current at the supply end in phase with $E_s$, in total amperes.

(1) *Circle Diagram.*

Describe a circle with centre $(a, b)$ and radius $c$, where

$$a = +\frac{1}{1000}\frac{E_s^2 \cdot R}{R^2 + X^2}$$

$$b = -\frac{1}{1000}\frac{E_s^2\, X}{R^2 + X^2}$$

$$\text{and} \quad c = +\frac{1}{1000}\frac{E\, E_s}{\sqrt{R^2 + X^2}}$$

The ordinates to this circle represent the reactive Kva., $\frac{E_s\, Q_s}{1000}$, at the generators.

(2) *Theoretical Limit of the Load in Kilowatts at the Supply End of the Line.*

The maximum load is

$$c + a$$

$$\text{or } \frac{1}{1000}\left(\frac{E\,E_s}{\sqrt{R^2 + X^2}} + \frac{E_s^2\,R}{R^2 + X^2}\right)$$

This is numerically greater than $c$ since $a$ is positive. It may be read from the circle diagram as it is the farthest distance to the right reached by the circle.

(3) *Reactive Kva. in the Line at Supply End.*

For a more precise value than that obtained from the circle diagram, solve the quadratic equation:

$$a_1 Q_s^2 - 2\, b_1 Q_s + c_1 = 0$$

where $a_1 = R^2 + X^2$

$$b_1 = -\,E_s X$$

and $c_1 = P_s^2\,(R^2 + X^2) - 2\, E_s P_s R + E_s^2 - E^2$

Then $Q_s = \dfrac{b_1 + \sqrt{b_1^2 - a_1 c_1}}{a_1}$

Use only the positive value of the radical. $Q_s$ is the reactive current, and $\dfrac{E_s\,Q_s}{1000}$, the reactive Kva., at the generators, for a given Kw. load, $\dfrac{E_s\,P_s}{1000}$, on the generators. When $Q_s$, as found above, is positive, it represents a leading load on the generators, and when it is negative, a lagging load.

(4) *Kva. at Generators.*

$$\frac{E_s \sqrt{P_s^2 + Q_s^2}}{1000}$$

$P_s$ is determined from the Kw. load at the generators, and $Q_s$ is either read from the circle diagram or calculated by means of (3).

(5) *Power-factor at the Generators.*

$$\frac{100\, P_s}{\sqrt{P_s^2 + Q_s^2}} \quad \text{per cent.}$$

(6) *Line Losses, in Kilowatts.*

$$\frac{(P_s^2 + Q_s^2)\, R}{1000}$$

(7) *Kw. at Receiver End.*

$$\frac{E_s P_s - (P_s^2 + Q_s^2)\, R}{1000}$$

(8) *Kva. at Receiver End.*

$$\frac{E \sqrt{P_s^2 + Q_s^2}}{1000}$$

(9) *Line Power-factor at Receiver End.*

$$100\, \frac{E_s P_s - (P_s^2 + Q_s^2)\, R}{E \sqrt{P_s^2 + Q_s^2}}$$

(10) *Reactive Kva. in the Line at Receiver End.*

$$\frac{E_s Q_s + (P_s^2 + Q_s^2)\, X}{1000}$$

When this quantity is positive, the reactive Kva. and the line power-factor are leading; and when it is negative, they are lagging.

(11) *Efficiency of the Transmission Line.*

$$100\,\frac{E_s P_s - (P_s^2 + Q_s^2)\,R}{E_s P_s}\ \text{per cent.}$$

(12) *Reactive Kva. of Synchronous Motors Required when Load Power-factor is cos θ.*

The reactive Kva. of the load are equal to the Kw. at receiver end, from (7), multiplied by $\frac{\sin\theta}{\cos\theta}$. To this result, treated as a positive quantity, add the reactive Kva. in the line at the receiver end, from (10), with due regard to the sign. This is a positive quantity when the line power-factor is leading. The sum is the reactive Kva. of synchronous motors required.

(13) *Amperes per Wire.*

As all the expressions for current are in total amperes, it may be noted that

$$\text{amperes per wire, three-phase,} = \frac{\text{total amperes}}{\sqrt{3}}$$

$$\text{amperes per wire, two-phase,} = \frac{\text{total amperes}}{2}$$

and amperes per wire, single-phase, = total amperes. For single-phase lines use 2 $R$ and 2 $X$ in place of $R$ and $X$.

(14) *Allowance for Extra Impedance.*

The resistance and reactance of the transformers and protective coils, referred to high tension, may be added to the resistance and reactance of the line. This involves no error or approximation at all when the electrostatic capacity of the line is not to be considered.

## TABLE III

*Formulas for Long Constant-Voltage Lines.*
*Conditions Given at Receiver End.*

These formulas give the results of the fundamental hyperbolic formulas as accurately as desired, if a sufficient number of terms of the convergent series is used.

*Conditions Given:*

$E$ = Voltage at receiver end.
$E_s$ = Voltage at supply end.
$Z = R + jX$.
$R$ = Resistance of one conductor.
$X$ = Reactance of one conductor.
$Y = G + jB$
$G$ = Leakage conductance of one conductor.
$B$ = Capacity susceptance of one conductor.
(See Tables VII–XVI, "Transmission Line Formulas.")
$P$ = In-phase current of the load, in total amperes.
$\cos\theta$ = Power-factor, lagging, of the load.

Find $E' + jE'' = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2.3.4} + \text{etc.}\right)$

and $\left(R' + jX'\right) = \left(R + jX\right)\left(1 + \frac{YZ}{2.3} + \frac{Y^2Z^2}{2.3.4.5} + \text{etc.}\right)$

(1) *Circle Diagram.*

Describe a circle with centre $(a'. b')$ and radius $c'$, where

$$a' = -\frac{E}{1000}\,\frac{E'R' + E''X'}{R'^2 + X'^2}$$

$$b' = +\frac{E}{1000}\,\frac{E'X' - E''R'}{R'^2 + X'^2}$$

$$\text{and}\quad c' = +\frac{E}{1000}\,\frac{E_s}{\sqrt{R'^2 + X'^2}}$$

Draw a straight line at an angle $\theta$ below the base line, where $\cos\theta$ is the P. F., lagging, of the load. By means of a pair of dividers, add the ordinates of the straight line to those of the circle, thus plotting the ellipse, giving the Kva. of synchronous motors required.

(2) *Theoretical Limit of Load, in Kilowatts.*

The maximum load is $c' + a'$

$$\text{or } \frac{E}{1000}\left(\frac{E_s}{\sqrt{R'^2 + X'^2}} - \frac{E' R' + E'' X'}{R'^2 + X'^2}\right)$$

This is numerically less than $c'$ since $a'$ is negative. It may be read from the circle diagram as it is the farthest distance to the right reached by the circle or the ellipse.

(3) *Reactive Kva. in the Line at Receiver End.*

For a more precise value than that obtained from the circle diagram, solve the quadratic equation:

$$a_2 Q^2 - 2\, b_2 Q + c_2 = 0$$

where $a_2 = R'^2 + X'^2$

$$b_2 = E' X' - E'' R'$$

and $c_2 = P^2 (R'^2 + X'^2) + 2\, P (E' R' + E'' X') + E'^2 + E''^2 - E_s^2$

$$\text{Then } Q = \frac{b_2 - \sqrt{b_2^2 - a_2 c_2}}{a_2}$$

Use only the negative value of the radical. $Q$ is the reactive current, and $\frac{EQ}{1000}$, the reactive Kva., in the line at the receiver, for a given Kw. load $\frac{EP}{1000}$. When $Q$, as found above, is positive, it represents leading reactive Kva. in the line at the receiver, and lagging, when it is negative.

(4) *Line Power-factor at Receiver End.*

$$\frac{100\,P}{\sqrt{P^2 + Q^2}} \quad \text{per cent.}$$

$P$ is determined from the Kw. load and $Q$ is either read from the circle diagram or calculated by means of (3).

(5) *Reactive Kva. of Load.*

$$\frac{E\,Q_l}{1000} = -\frac{E\,m\,P}{1000} \text{ (a negative quantity),}$$

where $Q_l$ is the reactive component of the load current, $\frac{E\,P}{1000}$ is the load in kilowatts at a lagging P. F. $\cos\theta$, and $m$ is a constant equal to $\frac{\sin\theta}{\cos\theta}$.

(6) *Reactive Kva. of Synchronous Motors.*

This may be read from the circle diagram, or it may be more accurately found by using the calculated value of $Q$ in the expression

$$\frac{E\,Q}{1000} + \frac{E\,m\,P}{1000} \quad \text{Kva.}$$

When the above expression is positive, the current in the synchronous motors is leading, necessitating strong excitation of the motors. When the expression is negative the current is lagging and the fields of the synchronous motors are under-excited.

(7) *Line Losses, in Kilowatts.*

Let
$$A = E' + P\,R' - Q\,X'$$
$$B = E'' + P\,X' + Q\,R'$$

and
$$C + j\,D = \left(P + j\,Q\right)\left(1 + \frac{Y\,Z}{2} + \frac{Y^2\,Z^2}{2\cdot3\cdot4} + \text{etc.}\right)$$

$$+ E Y \left(1 + \frac{Y Z}{2.3} + \frac{Y^2 Z^2}{2.3.4.5} + \text{etc.}\right)$$

where $Q$ is found either from the circle diagram or from (3).

$$\text{Line losses} = \frac{1}{1000} (A C + B D - E P) \text{ Kilowatts.}$$

(8) *Kw. at Generators.*

$$\frac{1}{1000} (A C + B D)$$

(9) *Kva. at Generators.*

$$\frac{E_s \sqrt{C^2 + D^2}}{1000}$$

(10) *Power-factor at Generators.*

$$\frac{100 (A C + B D)}{E_s \sqrt{C^2 + D^2}} \text{ per cent.}$$

(11) *Reactive Kva. at Generators.*

$$\frac{1}{1000} (A D - B C)$$

When this quantity is positive, the reactive Kva. and the generator P. F. are leading, and when it is negative, they are lagging.

(12) *Efficiency of the Transmission Line.*

$$\frac{100 E P}{A C + B D} \text{ per cent.}$$

(13) *Amperes per Wire.*

As all the expressions for current are in total amperes, it may be noted that

$$\text{amperes per wire, 3-phase,} = \frac{\text{total amperes}}{\sqrt{3}}$$

$$\text{amperes per wire, 2-phase,} = \frac{\text{total amperes}}{2}$$

and amperes per wire, single-phase, = total amperes.

For single-phase lines use $2R$ and $2X$ in place of $R$ and $X$, and use $\frac{1}{2}Y$ in place of $Y$

(14) *Allowance for Extra Impedance.*

Only a very slight error is introduced, for practical power-transmission lines, by adding the resistance and reactance of the transformers and protective reactance coils, referred to high tension, to the line resistance and reactance, when convergent series are used as in the preceding paragraphs. The exact solution would involve treating the impedances at the ends as separate sections of the line, each section to be calculated by itself. This method is long, and does not lend itself to the direct solution of the problems considered above. Such exactness of calculation also would not seem to be required for the usual accuracy of engineering problems in connection with power transmission.

## TABLE IV

### *Formulas for Long Constant-Voltage Lines*
### *Conditions Given at Supply End*

These formulas give the results of the fundamental hyperbolic formulas as accurately as desired, if a sufficient number of terms of the convergent series is used.

*Conditions given:*

$E$ = Voltage at receiver end.

$E_s$ = Voltage at supply end.

$Z = R + j X$.

$R$ = Resistance of one conductor.

$X$ = Reactance of one conductor.

$Y = G + j B$.

$G$ = Leakage conductance of one conductor.

$B$ = Capacity susceptance of one conductor.

(See Tables VII–XVI, "Transmission Line Formulas.")

$P_s$ = Current at the supply end in phase with $E_s$, in total amperes.

$\cos\theta$ = Power-factor, lagging, of the load.

Find $E_s' + j E_s'' = E_s\left(1 + \frac{YZ}{2} + \frac{Y^2 Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right)$

and $\left(R' + j X'\right) = \left(R + jX\right)\left(1 + \frac{Y\,Z}{2 \cdot 3} + \frac{Y^2\,Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \text{etc.}\right)$

(1) *Circle Diagram.*

Describe a circle with centre $(a', b')$ and radius $c'$, where

$$a' = + \frac{E_s}{1{,}000}\,\frac{E_s' R' + E_s'' X'}{R'^2 + X'^2}$$

$$b' = - \frac{E_s}{1{,}000}\,\frac{E_s'\,X' - E_s''\,R'}{R'^2 + X'^2}$$

$$\text{and } c' = + \frac{E_s}{1{,}000}\,\frac{E}{\sqrt{R'^2 + X'^2}}$$

The ordinates to this circle represent the reactive Kva., $\frac{E_s Q_s}{1{,}000}$, at the generators.

(2) *Theoretical Limit of the Load in Kilowatts at the Supply End of the Line.*

The maximum load is $c' + a'$

$$\text{or } \frac{E_s}{1{,}000}\left(\frac{E}{\sqrt{R'^2 + X'^2}} + \frac{E_s' R' + E_s'' X'}{R'^2 + X'^2}\right)$$

This is numerically greater than $c'$ since $a'$ is positive. It may be read from the circle diagram, as it is the farthest distance to the right reached by the circle.

(3) *Reactive Kva. in the Line at Supply End.*

For a more precise value than that obtained from the circle diagram, solve the quadratic equation:

$$a_2 Q_s^2 - 2\, b_2 Q_s + c_2 = 0$$

where $a_2 = R'^2 + X'^2$

$b_2 = -E_s' X' + E_s'' R'$

and $c_2 = P_s^2 (R'^2 + X'^2) - 2\, P_s (E_s' R' + E_s'' X') + E_s'^2 + E_s''^2 - E^2$

Then $Q_s = \dfrac{b_2 + \sqrt{b_2^2 - a_2 c_2}}{a_2}$

Use only the positive value of the radical. $Q_s$ is the reactive current, and $\frac{E_s Q_s}{1{,}000}$, the reactive Kva., at the generators, for a given Kw. load, $\frac{E_s P_s}{1{,}000}$, on the generators.

When $Q_s$, as found above, is positive, it represents a leading load on the generators, and when it is negative, a lagging load.

(4) *Kva. at Generators.*

$$\frac{E_s \sqrt{P_s^2 + Q_s^2}}{1,000}.$$

$P_s$ is determined from the Kw. load at the generators, and $Q_s$ is either read from the circle diagram or calculated by means of (3).

(5) *Power-factor at the Generators.*

$$\frac{100\, P_s}{\sqrt{P_s^2 + Q_s^2}} \text{ per cent.}$$

(6) *Line Loses in Kilowatts.*

$$\text{Let } F = E_s' - P_s R' + Q_s X'$$
$$H = E_s'' - P_s X' - Q_s R'$$

$$\text{and } M + j N = \left(P_s + j Q_s\right)\left(1 + \frac{Y Z}{2} + \frac{Y^2 Z^2}{2.3.4} + \text{etc.}\right)$$
$$- E_s Y \left(1 + \frac{Y Z}{2.3} + \frac{Y^2 Z^2}{2.3.4} + \text{etc.}\right)$$

where $Q_s$ is found either from the circle diagram or from (3).

$$\text{Line Losses} = \frac{1}{1,000}\left(E_s P_s - F M - H N\right) \text{ Kilowatts.}$$

(7) *Kw. at Receiver End.*

$$\frac{1}{1,000}\left(F M + H N\right)$$

(8) *Kva. at Receiver End.*

$$\frac{E \sqrt{M^2 + N^2}}{1,000}$$

(9) *Line Power-Factor at Receiver End.*

$$\frac{100\left(F M + H N\right)}{E \sqrt{M^2 + N^2}} \text{ per cent.}$$

(10) *Reactive Kva. in the Line at Receiver End.*

$$\frac{1}{1,000}\left(F N - H M\right)$$

When this quantity is positive, the reactive Kva. and the line P. F. are leading, and when it is negative, they are lagging.

(11) *Efficiency of the Transmission Line.*

$$100 \frac{F M + H N}{E_s P_s} \text{ per cent.}$$

(12) *Reactive Kva. of Synchronous Motors Required when Load Power-Factor is cos $\theta$.*

The reactive Kva. of the load are equal to the Kw. at receiver end, from (7), multiplied by $\frac{\sin \theta}{\cos \theta}$. To this result, treated as a positive quantity, add the reactive Kva. in the line at the receiver end, from (10), with due regard to the sign. This is a positive quantity when the line P. F. is leading. The sum is the reactive Kva. of synchronous motors required.

(13) *Amperes per Wire.*

As all the expressions for current are in total amperes, it may be noted that

$$\text{amperes per wire, 3-phase,} = \frac{\text{total amperes}}{\sqrt{3}}$$

$$\text{amperes per wire, 2-phase,} = \frac{\text{total amperes}}{2}$$

and amperes per wire, single-phase, = total amperes. For single-phase lines use a $2\,R$ and $2\,X$ in place of $R$ and $X$ and use $\frac{1}{2}\,Y$ in place of $Y$.

(14) *Allowance for Extra Impedance.*

Only a very slight error is introduced by adding the resistance and reactance of the transformers and reactance coils, referred to high tension, to the line resistance and reactance. See (14), Table III.

# TABLE V

## *Corona Limit of Voltage*

### Kilovolts at Sea Level for 3-phase Lines

| CABLES | | SPACING, IN FEET | | | | | |
|---|---|---|---|---|---|---|---|
| Size B. & S. or Circular Mils | Diameter, in Inches | 8 | 10 | 12 | 14 | 16 | 20 |
| 0 | 0.374 | 95 | 98 | 102 | 104 | 106 | 109 |
| 00 | 0.420 | 104 | 108 | 111 | 114 | 117 | 121 |
| 000 | 0.470 | 114 | 118 | 121 | 124 | 127 | 132 |
| 0,000 | 0.530 | 125 | 130 | 135 | 138 | 141 | 146 |
| 250,000 | 0.590 | 138 | 144 | 149 | 152 | 156 | 161 |
| 300,000 | 0.620 | ... | 151 | 156 | 161 | 165 | 171 |
| 350,000 | 0.679 | ... | 161 | 166 | 170 | 175 | 180 |
| 400,000 | 0.728 | ... | 171 | 176 | 180 | 185 | 192 |
| 450,000 | 0.770 | ... | 178 | 184 | 190 | 194 | 200 |
| 500,000 | 0.818 | ... | 188 | 194 | 199 | 205 | 210 |
| 800,000 | 1.034 | ... | ... | 234 | 241 | 244 | 256 |

### ALTITUDE CORRECTION FACTOR

| Altitude, in Feet | Correction Factor | Altitude, in Feet | Correction Factor |
|---|---|---|---|
| 0 | 1.00 | 5,000 | 0.82 |
| 500 | 0.98 | 6,000 | 0.79 |
| 1,000 | 0.96 | 7,000 | 0.77 |
| 1,500 | 0.94 | 8,000 | 0.74 |
| 2,000 | 0.92 | 9,000 | 0.71 |
| 2,500 | 0.91 | 10,000 | 0.68 |
| 3,000 | 0.89 | 12,000 | 0.63 |
| 4,000 | 0.86 | 14,000 | 0.58 |

To find the voltage for any altitude, multiply the voltage found in the table by the corresponding correction factor. For single-phase or 2-phase, find the 3-phase voltage above and multiply by 1.16.

# APPENDIX

## THE REACTANCE OF STRANDED CONDUCTORS

The reactance of a stranded conductor is appreciably different from that of a solid wire. For example, the reactance of No. 0000 cable of seven wires at 60 cycles and 18-inch spacing is 0.552 ohm per mile, while the reactance of a solid wire of the same sectional area is 0.560 ohm, and of a solid wire of the same diameter as the cable, 0.544 ohm. Thus the error in using a solid wire formula is in either case 1.5 per cent, which is too large to be neglected. This was explained by the author in "Transmission Line Formulas," Chapter XI, and formulas were given for 7-wire strands and 19-wire strands, which are the most common types of transmission-line conductors. The author has later published a more complete derivation of these formulas, with extensions applying to other types of conductors, in *The Electrical World*, April 19, 1913. The calculations in that article are here presented because the formulas are useful, not only in preparing tables of line constants, but in calculating the characteristics of transmission lines whose conductors have either an unusual size or number of wires.

The reactive voltage drop in an alternating-current circuit is due to alternating magnetic flux surrounding the conductors. Since flux which cuts both conductors does not produce a difference in voltage between them, and, therefore, does not produce any reactive drop in

the circuit, only that part of the flux must be considered which cuts one cable and not the other. It is necessary to calculate the voltage induced in each small wire of the cable $A$ (Fig. 12) by the flux in the space $s$ produced by each of the wires of cable $A$.

The radius of the cable is considered small compared with the distance $s$, since overhead lines are being considered. Round wires may be considered to be replaced by very small conductors located at the centres of the wires, so far as the inductance of one wire on another is

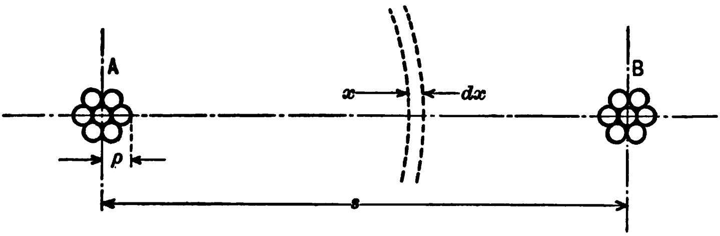


FIG. 12.—Single-phase Circuit.

concerned. The correction in inductance due to skin effect may be neglected, as it is very small for ordinary cables at 25 or 60 cycles, being only one-half of the skin effect correction in resistance.

Although the spiraling of the wires has a marked effect on the resistance of a strand (amounting to 1 or 2 per cent), there is practically no effect on the reactance, due to spiraling. The spiraling adds a component to the magnetic field produced by each wire, which would be in a plane parallel to the axis of the strand. The top wires cross the strand in the opposite direction to the bottom wires. The part of the above component which would be outside the strand would, therefore, produce

no effect on the reactance of the circuit. The effect of the part of the component inside the strand may be estimated by stating that it would increase the term 103 in the formula for 7 wires, by about 1 per cent, assuming that half of the wires are 2 per cent longer than the axis of the strand. Now, changing 103 to 104 in the formula would make a change in the final result of less than $1/10$ of 1 per cent, even for overhead lines of very narrow spacing. The above change would be negligible in the usual tables with three significant figures.

Suppose that the current in each small wire has unit value. Then the flux density due to that current, at a distance $x$ from the centre of the wire, is $\frac{2}{x}$.

If $t$ is the distance between any two wires of the cable, the required voltage induced in the second by the first, per centimetre, is

$$2\pi f \int_t^s \frac{2}{x}\,dx = 2\pi f \times 2 \log_e \frac{s}{t}$$
$$= 2\pi f M$$

where $f$ is the frequency, and $M$ is the inductance for the pair of wires.

The inductance of a wire due to its own current is given by the usual formula,

$$L_1 = 2 \log_e \frac{s}{r} + \frac{1}{2}$$

in which $r$ is the radius of the wire, and in which $1/2$ expresses the effect of the flux inside the wire.

The inductance per centimetre of cable of a single-phase circuit composed of 7-wire cables (Fig. 12) is porportional to

$$L' = 7\,L_1 + 12\,M_1 + 30\,M_2$$

where $L_1$ is the inductance of each wire due to its own current, $M_1$ is the inductance for pairs consisting of the centre wire and an outer wire, and $M_2$ is the average inductance for pairs consisting of two outer wires.

Then $$L_1 = 2 \log_e \frac{s}{r} + \tfrac{1}{2}$$

and $$M_1 = 2 \log_e \frac{s}{2r}$$

For the third term it is convenient to use the following theorem based on Cotes's theorem in trigonometry:

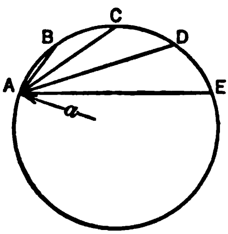


FIG. 13.—Geometrical Division of Circle.

If a circle of radius $a$ (Fig. 13) be divided into $m$ equal parts at $A, B, C, D$, etc., then $AB \cdot AC \cdot AD$ etc. (to $m-1$ factors) $= m\,a^{m-1}$*

Thus the mean value of log $t$

$$= \log \left(a\, m^{\frac{1}{m-1}}\right)$$

The average value of $M_2$ is, therefore.

$$2 \log_e \frac{s}{2 r 6^{\frac{1}{5}}}$$

$$\text{Therefore } L' = 98 \log_e \frac{s}{r} + \frac{7}{2} - 24 \log_e 2$$
$$- 60 \log_e 2 - 12 \log_e 6$$

In the above, the voltages induced in the seven wires have been added together and unit current has been

* C. E. Guye, "Comptes Rendus," Vol. CXVIII, p. 1329, 1894.
C. E. Guye, "L'Éclairage Électrique," Vol. III, p. 20, 1895.
E. B. Rosa, "Bulletin of the Bureau of Standards," Vol. IV, No. 2, p. 335, 1907.

assumed for each wire. Considering the cable as a single conductor, its inductance will be

$$L = \frac{1}{49} L'$$

Changing to practical units and referring to the maximum radius of the cable,

$$\rho = 3\, r,$$

we have $L = \left(103.3 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$

henrys per mile of conductor.

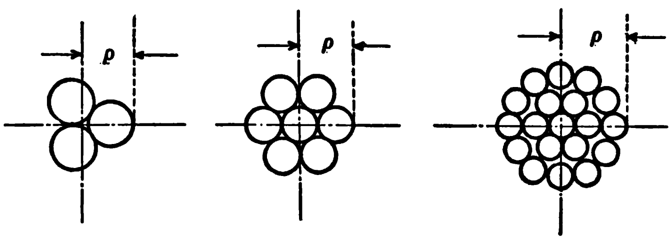


FIG. 14.

(*a*) 3-Wire Strand. (*b*) 7-Wire Strand. (*c*) 19-Wire Strand.

Applying the above method to a strand of $n$ wires, composed of $p$ layers around a centre wire (Fig. 14 *b* and c),

$$\begin{aligned} n &= 1 + 6 + 6 \times 2 + 6 \times 3 + \ldots\ldots + 6 \times p \\ &= 3\, p^2 + 3\, p + 1 \end{aligned}$$

and $L = 2 \log_e \frac{s}{\rho} + \frac{1}{2\, n} + 2 \log_e (2\, p + 1) -$

$$2\, \frac{(n - 1)}{n^2} \log_e 3$$

$$- \frac{24}{n^2} \left\{ 4 \log_e 2 + 2 \times 13 \log_e 4 + 3 \times 28 \log_e 6 + \ldots\ldots + p\,(n - 3\, p) \log_e (2\, p) \right\}$$

This gives,

for $n = 7$ and $p = 1$, $L = \left(103.3 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$

for $n = 19$ and $p = 2$, $L = \left(89.3 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$

for $n = 37$ and $p = 3$, $L = \left(85.1 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$

for $n = 61$ and $p = 4$, $L = \left(83.3 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$

and for solid wire,

$n = 1$ and $p = 0$, $L = \left(80.5 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$

A three-wire strand (Fig. 14 *a*) is a special case, since there is no central wire. The inductance is

$$L = 2 \log_e \frac{s}{\rho} + \frac{1}{6} + 2 \log_e \left(\frac{2\sqrt{3} + 3}{3}\right) - \frac{4}{3} \log_e 2 \text{ per cm.}$$

Therefore, $L = \left(125.2 + 741.13 \log_{10} \frac{s}{\rho}\right) 10^{-6}$ henrys per mile, where $\rho$ is the radius of the circumscribing circle of the cable.

The above equations may be put in the following simplified form, which is more convenient for calculating.

No. of wires in strand. Inductance per mile, in henrys.

3 . . . . . . . $L = 741.13 \log_{10} \frac{2.951\, s}{d} \times 10^{-6}$

7 . . . . . . . $L = 741.13 \log_{10} \frac{2.756\, s}{d} \times 10^{-6}$

19 . . . . . . . $L = 741.13 \log_{10} \frac{2.640\, s}{d} \times 10^{-6}$

37 . . . . . . . $L = 741.13 \log_{10} \frac{2.605\, s}{d} \times 10^{-6}$

$$61 \ldots\ldots\ldots L = 741.13 \log_{10} \frac{2.590\, s}{d} \times 10^{-6}$$

$$1 \text{ (Single wire) } L = 741.13 \log_{10} \frac{2.568\, s}{d} \times 10^{-6}$$

In the above, $d$ is the outside diameter of the cable, measured in the same units as $s$, the spacing between centres of the cables.

The significance of the above equations is best appreciated by consideration of the following examples of calculated reactance. The values tabulated are in ohms per mile at 60 cycles, where

$$\text{Reactance in ohms} = 2\,\pi \times 60\, L$$

Reactance per mile at 60 cycles of No. 0000 wire and cable, 211,600 circular mils.

| Feet Spacing | Solid Wire | 3-Wire Strand | 7-Wire Strand | 19-Wire Strand | 37-Wire Strand | 61-Wire Strand |
|---|---|---|---|---|---|---|
| 1½ | 0.560 | 0.550 | 0.552 | 0.546 | 0.544 | 0.543 |
| 6 | 0.728 | 0.718 | 0.721 | 0.714 | 0.712 | 0.711 |
| 12 | 0.812 | 0.802 | 0.805 | 0.799 | 0.797 | 0.796 |

The reactance of the cables is less than that of the wire, because the diameter is larger, though the sectional area is the same. The formulas, therefore, show that stranding reduces the reactance of a conductor of a certain cross-section by 1 or 2 per cent. As is well known, an increase of about the same percentage takes place in the resistance, due to stranding.